activities

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PYTHAGORAS ON PYRAMIDS

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Teacher's Guide

Grade Level: 8–12.

Materials: One copy of each sheet, a millimeter ruler, scissors, and a calculator with square-root function for each student.

Objectives: Using the Pythagorean theorem, we will suggest alternate strategies for finding altitudes of two different pyramids. Congruences of edges and faces will be noted. Areas and volumes will be computed.

Prerequisites: The students should know how to use the Pythagorean theorem prior to this lesson. Vocabulary such as *base*, *face*, *vertex*, and *height* should be familiar.

Directions:

1. Distribute sheets 1 and 2, a ruler, and scissors to each student.

2. Assist the class as necessary in assembling the pyramid model on sheet 1.

3. Have the students use the pyramid model to answer the questions on sheet 2. Round all computed lengths to the nearest millimeter.

4. Discuss the questions and answers on sheet 2.

5. Distribute sheet 3 to the class. Have the students assemble the second pyramid model, and assist as necessary. Put the following instructions on the blackboard or overhead projector:

> "Cut out the pyramid model and assemble it printed side out. Fold along all dashed lines. Cut along all dotted lines, including two slits on square *ABCD*. Do *not* cut out square *ABCD*. Insert TAB 1 into TAB 2, and then insert both TABs into slit *CD* on square *ABCD*. Insert TAB 3 into the slit on side $\overline{AD}$. TABs 1, 2, and 3 may be taped underneath."

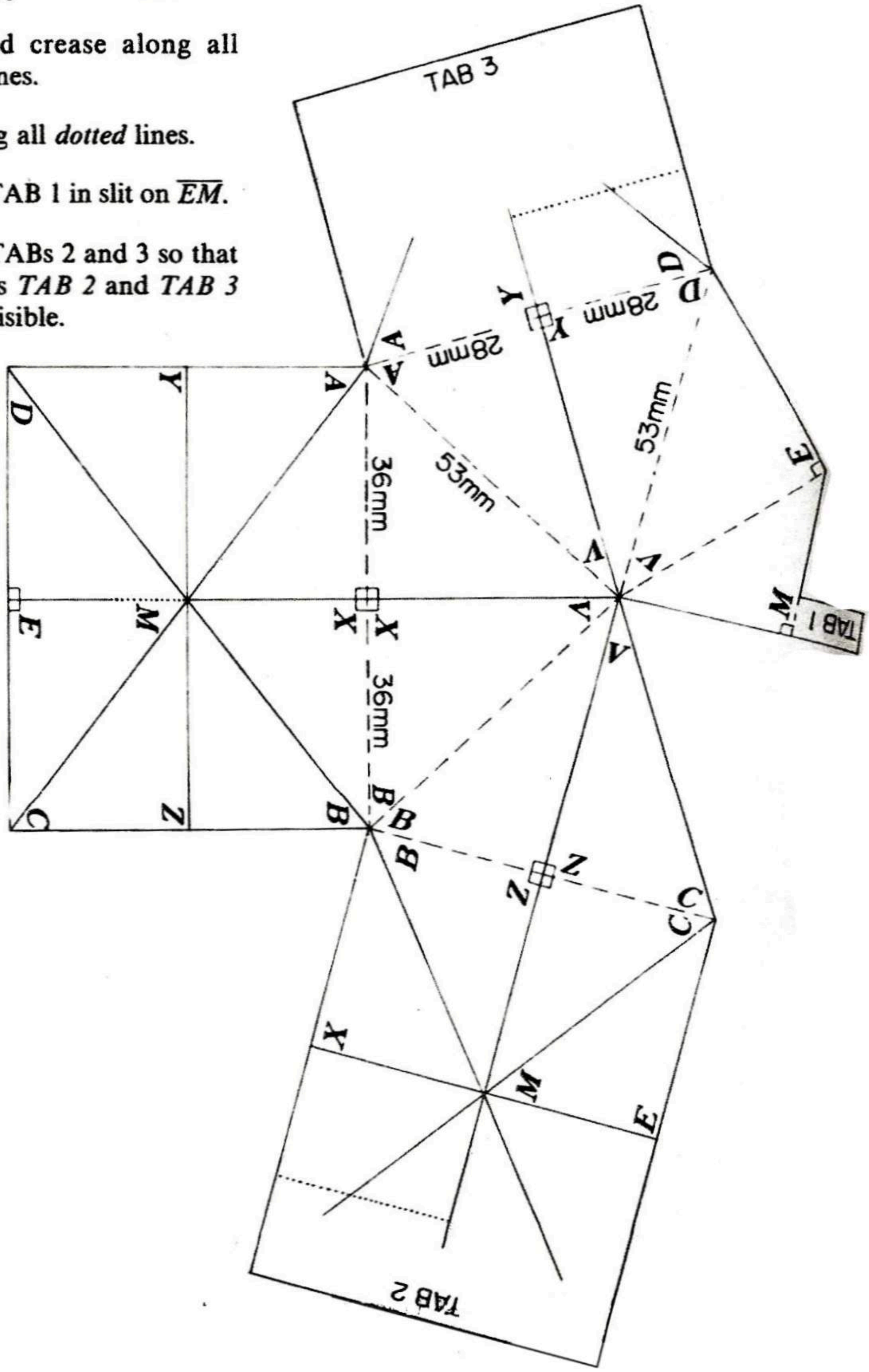
6. Have the students answer the questions on sheet 3.

7. Discuss sheet 3 with the class.

(Answers may be found on page 541.)

Instructions

1. Cut out pyramid model.
2. Fold and crease along all *dashed* lines.
3. Cut along all *dotted* lines.
4. Engage TAB 1 in slit on $\overline{EM}$.
5. Engage TABs 2 and 3 so that the words *TAB 2* and *TAB 3* are not visible.



These questions relate to the rectangular pyramid on sheet 1.
Triangle CDV should be considered a face of the pyramid.

1. List all the faces of the pyramid, indicating congruent faces.

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2. List all the edges of the pyramid and their lengths, indicating lengths that are congruent. ____

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3. Use the Pythagorean theorem to compute the slant heights (the altitudes of the sides from the base to the vertex) $\overline{VY}$ and $\overline{VX}$. Check your results by measuring.

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4. Compute the areas of each of the faces of the pyramid.

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5. The lateral area is the sum of the areas of the triangular faces of the pyramid. Compute the lateral area. ____

6. Compute the total surface area. ____

7. The Pythagorean theorem may be used to find the altitude $\overline{MV}$ of the pyramid. List all triangles that have $\overline{MV}$ as a side.

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8. Using length $\overline{EM}$, write an equation to find length MV.

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Using length $\overline{CM}$, write an equation to find length MV.

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9. Compute the length of $\overline{MV}$ and verify your results by measurement.

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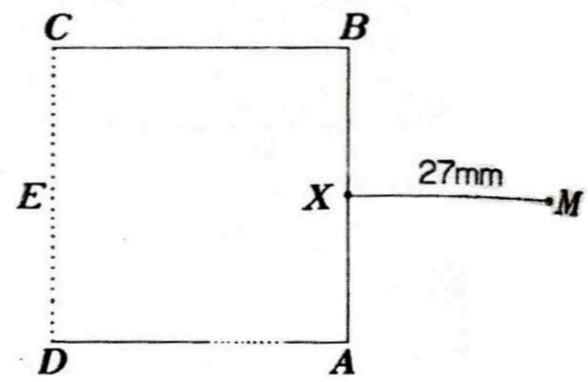
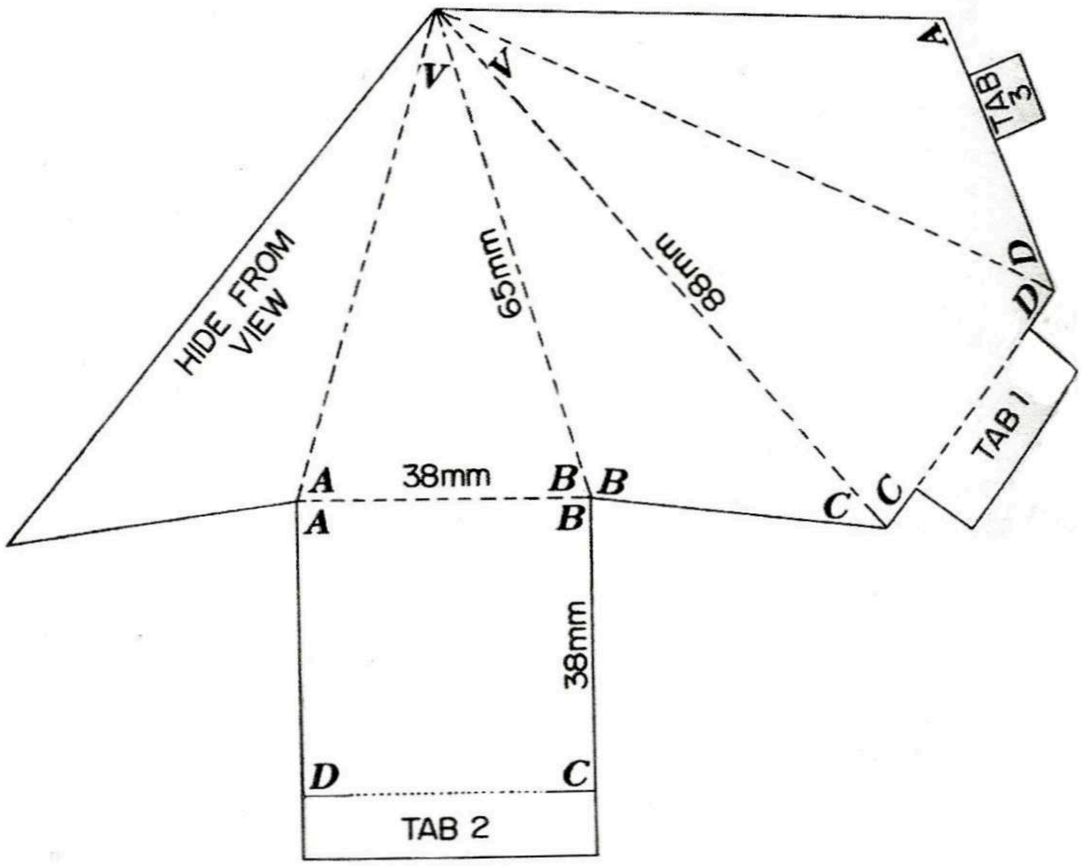
10. The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where h is the altitude and B is the area of the base. Compute the volume.

PATTERN FOR A SLANT PYRAMID — SHEET 3

Notes: Above this line cut only the dotted sections.

Base $ABCD$ is a square.





1. List all the faces of the pyramid, indicating congruent faces.

2. List all the edges and their lengths, indicating congruent edges.

3. Compute the slant heights $\overline{EV}$ and $\overline{VX}$.

4. The area of triangle BCV is 1121 mm^2. Compute the areas of each face, the lateral area, and the total surface area.

5. The point M is the foot of the perpendicular of the altitude $\overline{MV}$ of the pyramid. Write an equation to find length $\overline{MV}$ in terms of length $\overline{AV}$. Write an equation to find length $\overline{MV}$ in terms of length $\overline{EV}$.

6. Compute the length $\overline{MV}$ and verify your result by measuring.

7. What is the volume of the pyramid? How can this be verified experimentally?

Answers

Sheet 2

1. rectangle $ABCD$, $\triangle ADV \cong \triangle BCV$, $\triangle ABV \cong \triangle CDV$
2. $\overline{AD} \cong \overline{BC}$, 56 mm; $\overline{CD} \cong \overline{AB}$, 72 mm; $\overline{AV} \cong \overline{BV} \cong \overline{CV} \cong \overline{DV}$, 53 mm
3. $VY = 45$ mm; $VX = 39$ mm
4. area of the base $ABCD$ is 4032 mm^2; area of $\triangle ABV$ and $\triangle CDV$ are each 1404 mm^2; area of $\triangle ADV$ and $\triangle BCV$ are each 1260 mm^2
5. 5328 mm^2
6. 9360 mm^2
7. $\triangle EMV$, $\triangle CMV$, $\triangle MVZ$, $\triangle BMV$, $\triangle MVX$, $\triangle AMV$, $\triangle MVY$, $\triangle DMV$
8. $(\text{length } \overline{MV})^2 = (\text{length } \overline{EV})^2 - (\text{length } \overline{EM})^2$;

   $\text{length } \overline{MV} = \sqrt{(\text{length } \overline{CV})^2 - (\text{length } \overline{CM})^2}$
9. 27 mm
10. 36 288 mm^3

Sheet 3

1. square $ABCD$, $\triangle ABV$, $\triangle BCV \cong \triangle ADV$, $\triangle CDV$
2. $\overline{AV} \cong \overline{BV}$, 65 mm; $\overline{CV} \cong \overline{DV}$, 88 mm; $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$, 38 mm
3. length $\overline{EV}$ is 86 mm; length $\overline{VX}$ is 62 mm
4. area $\triangle ADV$ is 1121 mm^2; area $\triangle ABV$ is 1178 mm^2; area $\triangle CDV$ is 1634 mm^2; lateral surface area is 5054 mm^2; area square $ABCD$ is 1444 mm^2; total surface area is 6498 mm^2
5. $\text{length } \overline{MV} = \sqrt{(\text{length } \overline{VA})^2 - (\text{length } \overline{AM})^2}$;

   $\text{length } \overline{MV} = \sqrt{(\text{length } \overline{VE})^2 - (\text{length } \overline{EM})^2}$
6. 56 mm
7. 26 954 mm^3; by taping the seams of the pyramid, inverting it, and filling it with 26 954 mm^3 of a small, dry, solidlike salt, sugar, or sand