



pdf video

# Digital Manipulatives

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**"Tell me and I forget, teach me and I may remember,  
involve me and I learn."  
- Benjamin Franklin**



**S152: SATURDAY, 2:15-3:05 PM, CORONADO EF**

Hi. The session begins at 2:15.

Feel free to work/play before that.

Copies of the 5 spreadsheets are available on thumb drives in the room.

## First We'll "Play," Then We'll Work

Please introduce yourself to those around you.

Manipulative theory says: "Begin with free play & unstructured time before setting the stage for work" so we will do something like that.

Open the spread sheet just below and using the notes pictured on the spread sheet, see what you can do. Then we can better judge how to continue.

Don't worry about messing up. You can always download another spread sheet.

[Digital Manipulative Spread Sheet 1](#)

## Session Purpose and Objectives

This session and page is built for both those who know how to use concrete manipulatives and those who do not. If you do not know how to use a concrete manipulative, a digital manipulative is only a play thing. So we will consider a bit of theory first. This page is also written for those who have experience in cutting/selecting/pasteing and those who do not.

The time it takes to complete this work depends on one's skill set.

## Rational

# The Languages of the Math Classroom

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## MOTHER TONGUE & OTHER TONGUE(S)

← Most Sophisticated and also the Most Basic →

## MOSTLY MATH TONGUES

← Most Sophisticated, Most Basic →

| VERBAL / Auditory           | WRITTEN / Symbolic | PICTORIAL / Visual      | CONCRETE / Kinesthetic |
|-----------------------------|--------------------|-------------------------|------------------------|
| formal spoken mathematics   | written word       | DIGITAL MANIPULATIVE    | object                 |
| informal spoken mathematics | written symbol     | moving picture          | model                  |
| spoken symbol               | semisymbolic       | static picture          | manipulative/token     |
| symbol speak                | calculator symbol  | numeral                 |                        |
| calculatoreze/computereze   |                    | graph                   |                        |
| web speak                   |                    | nonverbal body language |                        |

## Manipulative and Languages

- Features of the Languages of the Math Classroom →
- Manipulative Use →

## Digital Manipulatives Purpose

Digital manipulatives were created to:

- Introduce in the concrete and debrief in the abstract.
- Increase the number of graphics used in class - speak pictorial more often.
- Increase visualization as a form of presentation, summary, debriefing.
- Create math experts, those who have enough facility in multiple languages to choose the language in which to work and communicate.
- Restate a mathematical idea in as many math class languages as possible for increased understanding on the part of both the speaker and the listener.
- Bring a cart load of manipulatives to class without having to bring a cart load of manipulatives to class.

## More Work and Play

Solve, & Create a Cartouche, & Add Vectors, & Write Digital Algebra Problems

[Digital Manipulative Spread Sheet 2](#)

Use multiple strips & fraction bars, & Add & Subtract on a Nomograph, & Assemble a jig saw puzzle, & Use Napier's Bones to Multiply, & Demonstrate that  $2/9 + 1/3$  is  $5/9$ .

[Digital Manipulative Spread Sheet 3](#)

Compute & Play on the 100s Boards

[Digital Manipulative Spread Sheet 4](#)

Calculi and Boards, & Egyptian Multiplication, & Slide Rules, & The Sine Law

- Chinese suan pan: 2/5, 2 over 5 (biquinary)
- Japanese soroban: 1/4, 1 over 4 (decimal)



[Digital Manipulative Spread Sheet 5](#)



[Multiply 25 x 12 by Doubling and Halving](#)

## Material Used Today

[Digital Manipulative Spread Sheet 1](#)

| Sheet             | Contents   |
|-------------------|--|
| select copy paste | Intro to Digit Manipulatives<br>This work from "Term Tiles & Tokens" |

@ [www.mathnstuff.com/math/algebra/ttloc.htm](http://www.mathnstuff.com/math/algebra/ttloc.htm) and  
 @ [www.termtiles.com](http://www.termtiles.com)

|                      |  |
|----------------------|--|
| problems 2 - 4       | Using Term Tiles to complete manipulative work.<br>This work from "Term Tiles & Tokens"<br>@ <a href="http://www.mathnstuff.com/math/algebra/ttloc.htm">www.mathnstuff.com/math/algebra/ttloc.htm</a>  |
| game for two players | The best middle school mental computation and vocabulary game ever.<br>This work from "A Game for Two Players"<br>@ <a href="http://www.mathnstuff.com/math/papers/games/42game.htm">www.mathnstuff.com/math/papers/games/42game.htm</a> and<br>@ <a href="http://www.mathnstuff.com/papers/games/42.xls">www.mathnstuff.com/papers/games/42.xls</a> |
| extra tiles & tokens | Storage. Because sometimes you need extra pieces.  |
| answers              | Answers to some problems.  |

### **Digital Manipulative Spread Sheet 2**

| Sheet          | Contents   |
|----------------|--|
| solve          | Solve $3 + 2x = 4x - 1$<br>This material is found<br>@ <a href="http://www.mathnstuff.com/math/algebra/tt22.htm">www.mathnstuff.com/math/algebra/tt22.htm</a> and<br><a href="http://www.mathnstuff.com/math/algebra/ttloc.htm">www.mathnstuff.com/math/algebra/ttloc.htm</a>  |
| hyro           | Create your own cartouche or write ancient Egyptian numbers or glyphs.<br>This is page: <a href="http://www.mathnstuff.com/papers/langu/hyrog1/hyro.xls">www.mathnstuff.com/papers/langu/hyrog1/hyro.xls</a> .   |
| vectors        | Add 4 @ $45^\circ$ and 6 @ $135^\circ$ by building the resultant vector on the rectangular plane.<br>This stuff is found<br>@ <a href="http://www.mathnstuff.com/math/xls/clocks.xls">www.mathnstuff.com/math/xls/clocks.xls</a> and<br>@ <a href="http://www.mathnstuff.com/math/spoken/here/2class/330/polrect.xls">www.mathnstuff.com/math/spoken/here/2class/330/polrect.xls</a> |
| write problems | Spread sheet completes polynomial computation and solves linear and quadratic equations.<br>This material is found<br>@ <a href="http://www.mathnstuff.com/math/algebra/ttloc.htm">www.mathnstuff.com/math/algebra/ttloc.htm</a>   |

### **Digital Manipulative Spread Sheet 3**

| Sheet                         | Contents   |
|-------------------------------|--|
| multiple strips/fraction bars | Write equivalent fractions. Add or subtract fractions.<br>See <a href="http://www.mathnstuff.com/math/spoken/here/2class/60/mult.htm">www.mathnstuff.com/math/spoken/here/2class/60/mult.htm</a> |
| nomograph                     | To add & subtract wholes, integers, fractions, decimals.   |



See [www.mathnstuff.com/math/spoken/here/2class/130/nomogrf/nomo.htm](http://www.mathnstuff.com/math/spoken/here/2class/130/nomogrf/nomo.htm) and [www.mathnstuff.com/math/spoken/here/2class/130/nomogrf/nomogrf.xls](http://www.mathnstuff.com/math/spoken/here/2class/130/nomogrf/nomogrf.xls) for more info.

|                        |   |
|------------------------|---|
| unit circle            | Assemble a unit circle jig saw puzzle.<br>Unit circle spread sheet found @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/330/gif/unit1.xls">www.mathnstuff.com/math/spoken/here/2class/330/gif/unit1.xls</a><br>Animated unit circle found at <a href="http://www.mathnstuff.com/math/spoken/here/2class/330/unit.htm">www.mathnstuff.com/math/spoken/here/2class/330/unit.htm</a><br>Manipulative master found at <a href="http://www.mathnstuff.com/math/spoken/here/2class/330/gif/unitc.gif">www.mathnstuff.com/math/spoken/here/2class/330/gif/unitc.gif</a>  |
| napier's bones         | Napier's Bones for multiplication, division, and taking roots. Problem 4: Multiply 2467 by 8.<br>Stored @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/60/napierb.xls">www.mathnstuff.com/math/spoken/here/2class/60/napierb.xls</a><br>Multiplication with Napier's Bones is @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/60/nbonesm.htm#mult">www.mathnstuff.com/math/spoken/here/2class/60/nbonesm.htm#mult</a><br>Division with Napier's Bones is @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/60/nbonesd.htm#divide">www.mathnstuff.com/math/spoken/here/2class/60/nbonesd.htm#divide</a><br>Root Extraction with Napier's Bones is @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/60/nbonesr.htm#sqrt">www.mathnstuff.com/math/spoken/here/2class/60/nbonesr.htm#sqrt</a> |
| thirds, sixths, ninths | Problem 5: Demonstrate that $2/9 + 1/3$ is $5/9$ .<br>Found @ <a href="http://www.mathnstuff.com/math/spoken/here/2class/70/frtiles/fract.xls">www.mathnstuff.com/math/spoken/here/2class/70/frtiles/fract.xls</a> with many other fraction bars.   |

#### **Digital Manipulative Spread Sheet 4**

| Sheet               | Contents  |
|---------------------|---|
| 100s                | regular 100s board  |
| negs                | positive, negative board                                      |
| multiples           | 12 copies of the 100s board colored with multiples of 2 to 13 |
| uncolored multiples | 12 copies of the 100s board                                   |

#### **Digital Manipulative Spread Sheet 5**

| Sheet   | Contents   |
|---------|--|
| calculi | Abaxes (Sand Tables), Reckoning Boards, Banks (Using Calculi)<br>See: <a href="http://www.mathnstuff.com/math/spoken/here/2class/40/40abacu/abacus.xls">www.mathnstuff.com/math/spoken/here/2class/40/40abacu/abacus.xls</a><br>See: <a href="http://www.mathnstuff.com/papers/amtj09.htm">www.mathnstuff.com/papers/amtj09.htm</a> for history & use of abacus.<br>See: The "units" worksheet of <a href="http://www.mathnstuff.com/math/xls/hands.xls">www.mathnstuff.com/math/xls/hands.xls</a> |

for a table which does not have powers of the same base.

Egyptian Mult Egyptian Multiplication  
 See: [www.mathnstuff.com/math/spoken/here/2class/40/40abacu/abacus.xls](http://www.mathnstuff.com/math/spoken/here/2class/40/40abacu/abacus.xls).  
 See: [www.mathnstuff.com/math/xls/xls.htm#digitalm](http://www.mathnstuff.com/math/xls/xls.htm#digitalm).  
 See: [www.mathnstuff.com/math/spoken/here/2class/60/egyptm.htm](http://www.mathnstuff.com/math/spoken/here/2class/60/egyptm.htm)  
 for Egyptian Computation.  
 See: [www.mathnstuff.com/math/spoken/here/2class/60/emulti.htm](http://www.mathnstuff.com/math/spoken/here/2class/60/emulti.htm),  
 Multiplication by Doubling and Halving (Mediation & Duplation) w/Calculi

## Additional Material Not Used Today

- [The Languages of the Math Classroom](#)
- [Digital Manipulatives on the Web Page Entitled Spread Sheets](#)
- [Math Tokens Home Page](#) Manipulatives Listed by Number Sense, ..., Geometry, ... Non-Math Stuff
- [Hold a shekel in your hands](#)
- [Divide the circle to compute its area](#)
- [Expand  \$\(x+y\)^2\$](#)
- ["Roll" a die or pair of dice](#)
- [Play addition rummy with a deck of cards](#)

## Additional Material Not On This Site

The International Slide Rule Museum

- [Slide Rule History](#)
- [Simulated Pickett N909-ES Slide Rule](#), © 2005, Derek Ross



[www.mathnstuff.com/papers/dmanip1.htm](http://www.mathnstuff.com/papers/dmanip1.htm) © 12/2017, 11/2018, 9/2023,

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