



Hypothesis Tests

Read [Hypothesis Tests](#) (How to Complete a Hypothesis Test)

[Table of Contents for CALCULATORing](#)
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[TI83-84 STATISTICS and DISTRIBUTION Menus](#)

Test	significance test for:	test statistic (observed"new") - (expected"old") (test) = $\frac{\text{---}}{\text{---}}$ (standard error)
Z-Test	1 mean, normal, large sample size	$z_{\text{test}} = (\bar{x} - \mu) / (\sigma / \sqrt{n})$
T-Test	1 mean, small sample, sigma unknown	$t_{\text{test}} = (\bar{x} - \mu) / (s / \sqrt{n})$
2 Sample Z-Test	2 means	$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
2 Sample T-Test	2 INDEPENDENT means	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
2 Sample T-Test	2 DEPENDENT means	$t = \frac{\frac{\sum (x_1 - x_2)}{n} - \mu_{(x_1 - x_2)}}{\sqrt{\frac{n \sum (x_1 - x_2)^2 - (\sum (x_1 - x_2))^2}{n(n-1)}}}$ \sqrt{n}
ANOVA Test	3 or more means using variances	

$$F = \frac{S_{between}^2}{S_{within}^2}$$

1 Proportion Z-Test 1 proportion

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

2 Proportion Z-Test 2 proportions

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

χ^2 Test

discrete (catagories)
Independence of Proportion
Is the preference independent of the group?

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

CREATE an EXPECTED (row,column) distribution.

χ^2 Test

discrete (catagories)
Homogeneity of Proportions
Are preferences of subgroups the same?

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

CREATE an EXPECTED (row,column) distribution.

Linear Regression T-Test 1 correlation coefficient, between x and y

$$t = r \sqrt{\frac{n-2}{1-r^2}}$$

χ^2 GOF Test

Goodness of Fit, match
"test" distribution to another distribution

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

χ^2 GOF Test

Normality, match
"test" distribution to a normal distribution

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

CREATE an EXPECTED distribution.

χ^2 Test

1 Variance

$$\chi^2_{test} = [(n-1)s^2]/\sigma^2$$

Two Sample F Test 2 variances

$$F = \frac{S_{larger\ variance}^2}{S_{smaller\ variance}^2}$$

Sign [Test](#)

1 median
or 1 pair, as in (After) - (Before) > 0
not very sensitive a test

$$z_{sign} = \frac{x + .5 - n/2}{\sqrt{n/2}}$$

$$n = x^+ + x^-, \quad n > 25$$

Wilcoxon
Mann-Whitney
Rank Sum [Test](#)

2 distributions
for symmetry, center, distribution of ranks
with averaged ranks awarded to tied data
points

$$z_{Wilcoxon_{rank\ sum}} = \frac{R - \mu_R}{\sigma_R}$$

Wilcoxon
Signed-Rank [Test](#)

median of 2 distributions
before-after
small sample size, not nec. normal

$$z = \frac{W - 0.5}{\sqrt{\frac{n(n+1)(2n+1)}{6}}}$$

$$W = \left| \sum [\text{sgn}(x_2 - x_1) \cdot R] \right|$$

Z-Test Uses the Normal or Gaussian Probability Density Function or Distribution

This tests to see if evidence exists that the real mean is not the stated mean.

Needed:

$$n \geq 30$$

or, distribution is normal and standard deviation is known.

The null hypothesis is always:

$$H_0: \mu = (\text{stated mean of the population})$$

The alternate hypothesis is always one of these:

$$H_1: \mu < (\text{stated mean of the population})$$

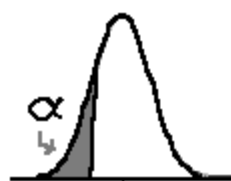
$$H_1: \mu = (\text{stated mean of the population})$$

$$H_1: \mu > (\text{stated mean of the population})$$

Test statistic:

$$z_{test} = (\bar{x} - \mu) / (\sigma / \sqrt{n})$$

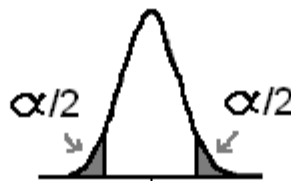
Levels of confidence:



$$H_0: \mu = k$$

$$H_1: \mu < k$$

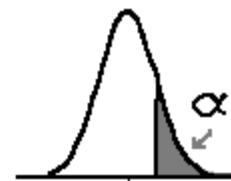
α	z critical
0.10	-1.28
0.05	-1.65
0.01	-2.33



$$H_0: \mu = k$$

$$H_1: \mu \neq k$$

α	z critical
0.10	± 1.65
0.05	± 1.96
0.01	± 2.58



$$H_0: \mu = k$$

$$H_1: \mu > k$$

α	z critical
0.10	1.28
0.05	1.65
0.01	2.33

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then 7:Z-Interval...

To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then 1:Z-Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

T-Test

This tests to see if evidence exists that the real mean is not the stated mean.

Needed:

approximately normal distribution,
unknown population standard deviation.

$n < 30$.

the larger the sample size, the closer the distribution approaches normal.
degrees of freedom, symbolized d.f., equals $n-1$.

The null hypothesis is always:

$$H_0: \mu = (\text{stated mean of the population})$$

The alternate hypothesis is always one of these:

$$H_1: \mu < (\text{stated mean of the population})$$

$$H_1: \mu = (\text{stated mean of the population})$$

$$H_1: \mu > (\text{stated mean of the population})$$

Test statistic:

$$t_{\text{test}} = (\bar{x} - \mu) / (s / \sqrt{n})$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.
or by calculator

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then 7:TInterval...

To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then 2:T-Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

1 Proportion Z-Test

Used to test claims concerning percents, probabilities, proportions. as in "70% of the population," "42% of the products"

Needed:

p , the population proportion, the stated proportion

$\hat{p} = x/n$, the sample proportion

$np \geq 5, nq \geq 5$, where $q = 1 - p$

$$\mu = n \cdot p \quad \sigma = \sqrt{n \cdot p \cdot q}$$

The null hypothesis is always:

$H_0: p =$ (stated proportion), may be symbolized as p_0

The alternate hypothesis is always one of these:

$H_1: p \neq$ (stated proportion)

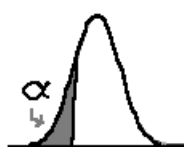
$H_1: p <$ (stated proportion)

$H_1: p >$ (stated proportion)

Test statistic:

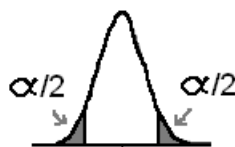
$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Levels of confidence: as above z-test



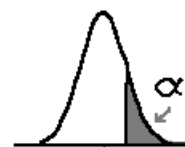
$H_0: \mu = k$
 $H_1: \mu < k$

α	z critical
0.10	-1.28
0.05	-1.65
0.01	-2.33



$H_0: \mu = k$
 $H_1: \mu \neq k$

α	z critical
0.10	± 1.65
0.05	± 1.96
0.01	± 2.58



$H_0: \mu = k$
 $H_1: \mu > k$

α	z critical
0.10	1.28
0.05	1.65
0.01	2.33

or by calculator

To Run a Calculator Confidence Interval to Test: (for 2-tail test)
[STAT], then [TESTS], then A:1-PropZInterval...

To Run a Calculator Hypothesis to Test:
[STAT], then [TESTS], then 5:1-PropZTest...

One page [Summary](#), sample problem, and a [spreadsheet](#).

Linear Regression T-Test

This tests to see if evidence exists that there is a correlation between the independent variable, x , and the dependent variable, y . The population correlation coefficient, ρ ,

symbol ρ , is the strength of the linear relationship between x and y.
 The relationship ranges between
 a strong negative relationship (as x increases, y decreases)
 to a weak relationship, to no relationship (0),
 to a weak positive relationship (as x increases, y increases),
 to a strong positive relationship.
 $-1 \leq r \leq 1$

Needed:

r, the sample correlation coefficient is

$$r = \frac{n(\sum(xy)) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

d.f. = n-2

The null hypothesis is always:

$$H_0: \rho = 0$$

The alternate hypothesis is always one of these:

$$H_1: \rho < 0 \quad H_1: \rho \neq 0 \quad H_1: \rho > 0$$

Test statistic:

$$t = r \sqrt{\frac{n-2}{1-r^2}}$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then G:LinRegTInt...

To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then F:LinRegTTest...

One page [Summary](#), sample problem, and a [spreadsheet](#).

2 Sample Z-Test

This tests to see if evidence exists that there is a difference between 2 means.

Needed:

two populations with known standard deviations.

independence between samples.

normal or approximately normal distributions if sample sizes are less than 30.

The null hypothesis is always:

$$H_0: \mu_1 = \mu_2 \text{ or } H_0: \mu_1 - \mu_2 = 0$$

The alternate hypothesis is always one of these:

$$H_1: \mu_1 \neq \mu_2 \text{ or } H_1: \mu_1 - \mu_2 \neq 0$$

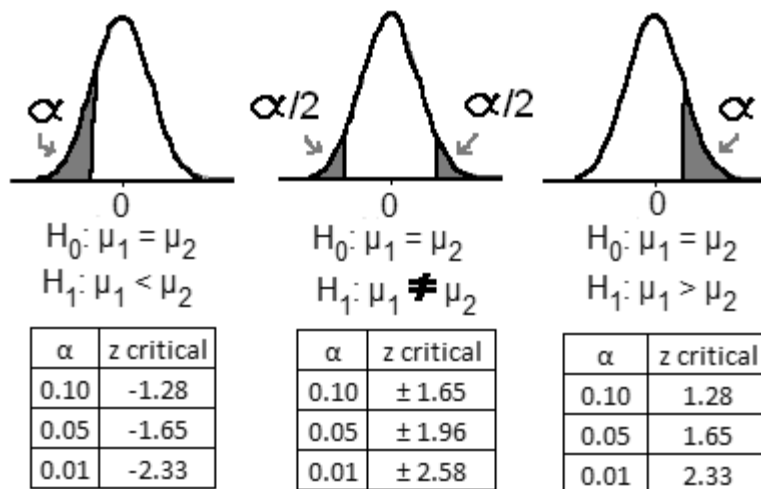
$$H_1: \mu_1 > \mu_2 \text{ or } H_1: \mu_1 - \mu_2 > 0$$

$$H_1: \mu_1 < \mu_2 \text{ or } H_1: \mu_1 - \mu_2 < 0$$

Test statistic: **[Because the null hypothesis is $\mu_1 - \mu_2 = 0$, the numerator is simplified!]**

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Levels of confidence:



or by calculator

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then 9:2SampZInterval...

To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then 3:2SampZ-Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

2 Sample T-Test, Independent Samples

This tests to see if evidence exists that there is a difference between 2 means.

Needed:

- two populations, means and standard deviation need not be known.
- sample standard deviations must be known.
- independence between samples and are not matched or paired,
- normal or approximately normal distributions if sample sizes are less than 30.

The null hypothesis is always:

$$H_0: \mu_1 = \mu_2 \text{ or } H_0: \mu_1 - \mu_2 = 0$$

The alternate hypothesis is always one of these:

$$H_1: \mu_1 \neq \mu_2 \text{ or } H_1: \mu_1 - \mu_2 \neq 0$$

$$H_1: \mu_1 > \mu_2 \text{ or } H_1: \mu_1 - \mu_2 > 0$$

$$H_1: \mu_1 < \mu_2 \text{ or } H_1: \mu_1 - \mu_2 < 0$$

Test statistic: [Because the null hypothesis is $\mu_1 - \mu_2 = 0$, the numerator is simplified!]

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Levels of confidence:

$$d.f. = \frac{\frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$$

d.f. smaller of n_1-1 and n_2-1
 or by weighted degrees of freedom
 as shown at right and in [spreadsheet](#).

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then 0:2-SampTInt...

To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then 4:2SampTTest...

One page [Summary](#), sample problem, and a [spreadsheet](#).

2 Sample T-Test, Dependent Samples, Paired, Before/After

$$\bar{D} = \frac{\sum (x_1 - x_2)}{n}$$

This tests hypothesises about the mean of the difference of paired samples. It's great for before and after samples. Clearly the samples are not independent, but paired one-to-one to create differences which become the sample statistics.

This mean one is testing to see if there is a statistically significant difference before and after some treatment or event.

Needed:

$D = x_1 - x_2$ for every pair of n samples

they are n matched or paired samples, as in before and after samples

each population is normal or approximately normal

d.f. = $n-1$

The null hypothesis is always: $H_0: \mu_D = 0$ (, or some constant)

The alternate hypothesis is always one of these:

$H_1: \mu_D < 0$ (, or some constant)

$H_1: \mu_D \neq 0$ (, or some constant)

$H_1: \mu_D > 0$ (, or some constant)

Test statistic: **[Because the null hypothesis is $\mu_1 - \mu_2 = 0$ or some constant, the numerator may be simplified!]**

$$t = \frac{\frac{\bar{D} - \mu_D}{\sqrt{\frac{n \sum D^2 - (\sum D)^2}{n(n-1)}}}}{\sqrt{n}} \quad t = \frac{\frac{\frac{\sum (x_1 - x_2)}{n} - \mu_{(x_1 - x_2)}}{\sqrt{\frac{n \sum (x_1 - x_2)^2 - (\sum (x_1 - x_2))^2}{n(n-1)}}}}{\sqrt{n}}$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.
 To Run a Calculator Hypothesis to Test:
 Create a list of the D distribution using $D = x_1 - x_2$ for every pair of n samples
 then [STAT], then [TESTS], then 2:T-Test... on this D distribution
 One page [Summary](#), sample problem, ([calculator version](#)), and a [spreadsheet](#).

2 Proportion Z-Test

Used to test claims concerning 2 percents, probabilities, proportions, as in "70% of males," vs "42% of females."

Needed:

samples are independent

$$n_1 p_1 \geq 5 \quad n_1 q_1 \geq 5 \quad n_2 p_2 \geq 5 \quad n_2 q_2 \geq 5$$

The null hypothesis is always:

$$H_0: p_1 = p_2$$

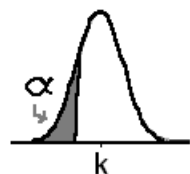
The alternate hypothesis is always one of these:

$$H_1: p_1 > p_2 \quad H_1: p_1 \neq p_2 \quad H_1: p_1 < p_2$$

Test statistic:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad z = \frac{\left(\frac{x_1}{n_1} - \frac{x_2}{n_2}\right) - (p_1 - p_2)}{\sqrt{\left(\frac{x_1 + x_2}{n_1 + n_2}\right)\left(1 - \frac{x_1 + x_2}{n_1 + n_2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

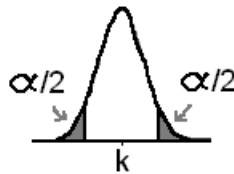
Levels of confidence:



$$H_0: \mu = k$$

$$H_1: \mu < k$$

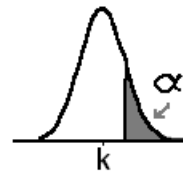
α	z critical
0.10	-1.28
0.05	-1.65
0.01	-2.33



$$H_0: \mu = k$$

$$H_1: \mu \neq k$$

α	z critical
0.10	± 1.65
0.05	± 1.96
0.01	± 2.58



$$H_0: \mu = k$$

$$H_1: \mu > k$$

α	z critical
0.10	1.28
0.05	1.65
0.01	2.33

To Run a Calculator Confidence Interval to Test: (for 2-tail test)

[STAT], then [TESTS], then B:2-PropInterval...

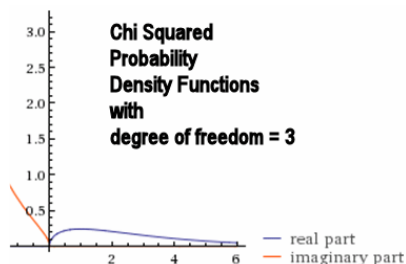
To Run a Calculator Hypothesis to Test:

[STAT], then [TESTS], then 6:2PropZ-Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

χ^2 Test for A Single Variance

This tests to see if evidence exists that



the real variance is not the stated variance.
It's a test for consistency, hoping for little, but enough, variation.

Needed:

- normal population
- random sample of n independent observations
- population standard deviation and sample standard deviation
- degrees of freedom, d.f. = n-1

The null hypothesis is always:

$$H_0: \sigma^2 = (\text{stated population variance})$$

The alternate hypothesis is always one of these:

$$H_1: \sigma^2 < (\text{stated population variance})$$

$$H_1: \sigma^2 \neq (\text{stated population variance})$$

$$H_1: \sigma^2 > (\text{stated population variance})$$

Test statistic:

$$\chi^2_{\text{test}} = [(n-1)s^2]/\sigma^2$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.

CAN'T Run a Calculator Hypothesis to Test w/o a program

CAN Use [CATALOG] and χ^2 cdf(in a program

8: χ^2 cdf(lower bound,upperbound,degrees of freedom)

One page [Summary](#), sample problem, and a [spreadsheet](#).

χ^2 Test for Independence of Proportions

This tests to see if evidence exists that the preferences of the groups is independent of the group making the choice.

Needed:

- Radom sample of full population is sorted by groups and their preferences, usually sorted into rows and columns and called a contingency table.
- each cell of table must be 5 or greater or groups/preferences must be pooled
- degrees of freedom, d.f. = ((rows)-1)((columns)-1)

The null hypothesis is always:

H_0 : The preference is independent of the group.

The alternate hypothesis is always:

H_1 : The preference is dependent on the group.

Test statistic: EXPECTED value table must be generated first using the ratio:(row sum)(column sum)/((grand sum) for each cell in table.

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.

To Run a Calculator Hypothesis Test:

[MATRIX] then edit OBSERVED and EXPECTED matrices, then [STAT], then [TESTS], then C: χ -Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

χ^2 Test for Homogeneity of Proportions

This tests to see if evidence exists that the one or more of the subgroup in a population has a different distribution of proportions than the other subgroups.

Needed:

each subgroup is randomly sampled,
 preferences are sorted into rows and columns called a contingency table
 each cell of table must be 5 or greater or groups/preferences must be pooled
 degrees of freedom, d.f. = ((rows)-1)((columns)-1)

The null hypothesis is always:

H_0 : $P_{\text{subgroup1}} = P_{\text{subgroup2}} = P_{\text{subgroup3}} = \dots = P_{\text{last subgroup}}$

The alternate hypothesis is always:

H_1 : One or more of the ps is different.

Test statistic: EXPECTED value table must be generated first using the ratio:

(row sum)(column sum)/(grand sum) for each cell in the contingency table.

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

Levels of confidence:

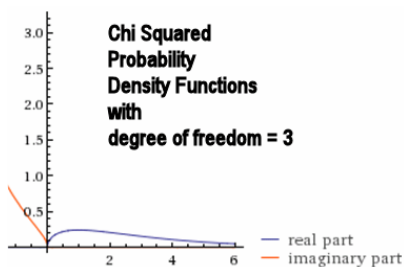
found in table organized by 1-tail or 2-tail and d.f.

To Run a Calculator Hypothesis Test:

[MATRIX] then edit OBSERVED and EXPECTED matrices, then [STAT], then [TESTS], then C: χ -Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

χ^2 Goodness of Fit Test



This tests to see if the observed result compares favorably to the theoretical (or prior) results. It also tests if a distribution is normal.

The test looks at, all at once, how the frequency distributions of observed categories compares to their expected frequencies.

It answers the question: "Does the observed result fit the pattern that was predicted?"

Recall that a completely random pattern would

have about equal frequencies in each category.

Needed:

random sample of size n where $n/(\text{number of categories}) \geq 5$

O = the observed frequency in each category

E = the expected frequency in each category, WHICH IS...

based on a random distribution: $E = n/(\text{number of categories})$

based on a prior distribution: $E = (\text{prior category percent})(n)$

degrees of freedom, d.f. = (number of categories) - 1

The null hypothesis is:

H_0 : The distribution is the same as the published distribution.

(It's a reasonable fit.)

H_0 : The distribution is normally distributed.

The alternate hypothesis is:

H_1 : The distribution is not the same as the published distribution.

(It's not a really good fit.)

H_1 : The distribution is not normally distributed.

Test statistic:

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

Levels of confidence:

found in table organized by 1-tail or 2-tail and d.f.

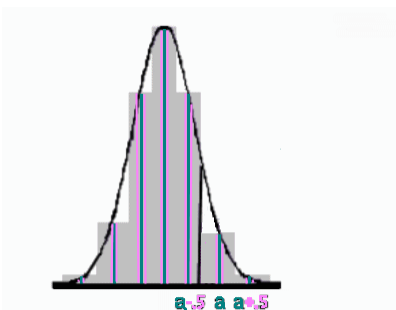
To Run a Calculator Hypothesis Test:

Enter the OBSERVED *frequencies* in one list and EXPECTED in a matching list.

[STAT], then [TESTS], then D: χ^2 -Test...

One page [Summary](#), sample problem, and a [spreadsheet](#).

χ^2 Test for Normality



$$P(x=a) = P(x < (a + .5)) - P(x < (a - .5))$$

The [Chi Square Goodness of Fit Test](#) is here used to see if a distribution is normal.

Three steps are required.

1st: Compute the mean and standard deviation of the OBSERVED data (green).

2nd: Create an EXPECTED distribution using the normal cumulative distribution function and the OBSERVED mean and st.dev. (blue).

3rd: Run a GOF test using the OBSERVED and EXPECTED distribution frequencies (purple).

The p in the test below is .619. The null hypothesis is confirmed. The OBSERVED distribution is normal.

OBSERVED DISTRIBUTION

mean is **110.00**
 standard deviation is **34.66**
 variance **1201.50**

mark, z	O freq, f	fx	fx ²
1	50	100	5000
2	90	300	27000
3	130	300	39000
4	170	100	17000

mean is Σfx is **88000**
110 Σfx^2 is **7.7E+09**
 Σfx^2 is **1.1E+07**

800 n
799 n-1 is

7.7E+08 $n(\Sigma fx^2) - (\Sigma fx)^2$
639200 $n(n-1)$

1201.5 $[n(\Sigma fx^2) - (\Sigma fx)^2] / [n(n-1)]$
 the variance
34.6627 $\sqrt{\text{variance}}$
 the standard deviation.

$$s^2 = \frac{n(\sum fx^2) - (\sum fx)^2}{n(n-1)}$$

EXPECTED DISTRIBUTION OBSERVED statistics.

mean is **110.00**
 standard deviation is **34.66**
 variance **1201.50**

	lower	upper	mark, z	E%	Efreq
1	30	70	50	0.1138	91.0030
2	70	110	90	0.3757	300.5964
3	110	150	130	0.3757	300.5964
4	150	190	170	0.1138	91.0030

	observed	expected	O - E	(O-E)^2	(O-E)^2/E
1	100	91.00	8.9970	80.9466	0.8895
2	300	300.60	-0.5964	0.3557	0.0012
3	300	300.60	-0.5964	0.3557	0.0012
4	100	91.00	8.9970	80.9466	0.8895

d.f is **3.00** chi square **1.7814**

COMPLETE A GOF TEST FOR NORMALITY OF DISTRIBUTION
 H₀: Observed distribution is normal.
 H₁: Observed distribution is not normal.

chisquaredGOFtest
 p is **0.619** $\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right]$

One page [Summary](#) and a [spreadsheet](#).

Two Sample F-Test

This tests to see if evidence exists that two variances differ.
 Needed:

- 2 normal distributions
- independent samples
- sample standard deviations
- number in each sample
- d.f.n. (numerator) = n_{numerator}-1
- d.f.d. (denominator) = n_{denominator}-1

The null hypothesis is always:

$$H_0: \sigma_{\text{larger}}^2 = \sigma_{\text{smaller}}^2$$

The alternate hypothesis is always one of these:

$$H_1: \sigma_{\text{larger}}^2 < \sigma_{\text{smaller}}^2$$

$$H_1: \sigma_{\text{larger}}^2 \neq \sigma_{\text{smaller}}^2$$

$$H_1: \sigma_{\text{larger}}^2 > \sigma_{\text{smaller}}^2$$

Test statistic:

$$F = \frac{S_{\text{larger variance}}^2}{S_{\text{smaller variance}}^2}$$

Levels of confidence:

found in tables organized by d.f.n and d.f.d.

To Run a Calculator Confidence Interval to Test: (for 2-tail test)
 [STAT], then [TESTS], then 7:TInterval...
 To Run a Calculator Hypothesis to Test:
 [STAT], then [TESTS], then 2:T-Test...
 One page [Summary](#), sample problem, and a [spreadsheet](#).

ANOVA -- Analysis of Variance, One-Way, Single Factor

This tests to see if evidence exists that means of 3 or more groups differ when the variances are equal.
 Needed:

- population must be normal or near normal
- independent samples
- variances equal
- $N = n_1 + n_2 + n_3 + \dots + n_k$, k = the number of groups
- d.f.numerator = $k - 1$
- d.f.denominator = $N - k$
- The grand mean, GM, is the mean of all data.

The null hypothesis is always:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$$

The alternate hypothesis is always:

H_1 : One or more of the means is not equal to the others

Test statistic:

The F test statistic,

the sum of the squares between groups is SS_B , $(SS_B)/(k-1) = MS_B = s_{between}^2$

the sum of the squares within groups is SS_W , $(SS_W)/(N-k) = MS_W = s_{within}^2$

$$F = \frac{s_{between}^2}{s_{within}^2} \quad s_{between}^2 = \frac{\sum [n_i (\bar{x}_i - \bar{x}_{GM})^2]}{k - 1} \quad s_{within}^2 = \frac{\sum [(n_i - 1) s_i^2]}{\sum (n_i - 1)}$$

Levels of confidence:

found in table organized by alpha, d.f.numerator, d.f.denominator

To Run a Calculator Hypothesis to Test:

[STAT], then [EDIT] to enter lists, then [STAT], [TESTS], then H:ANOVA(L₁, L₂, L₃, ... ,L_{last})

Page one of [Summary](#), page 2 of [Summary](#), sample problem, and a [spreadsheet](#).

Sign Test

This is really a binomial (+ or - response) test to see if evidence exists that a median is the hypothesized median (single sample) or to see if a difference exists in paired data. If n is less than 26, a formula is not used, just a count and compare procedure. If n is greater than 25, a z-test is used. In either case, the smaller of the number of data points above the median and number of data points below the mean is used to compute the test statistic.

Needed:

- a sample of 100 to do the job a sample of 30 does w/a more efficient test
- may be nonparametric (not defined mean, standard deviation, etc.)

random sample

use the raw data to find the "sign" of [(data)-(stated median)]

Compute each [(data)-(stated median)] or [(After)-(Before)] first to determine $n = x^+ + x^-$!!!

The null hypothesis always:

$$H_0: \text{median} = (\text{stated median}) \quad \text{or} \quad H_0: (\text{After}) - (\text{Before}) = 0$$

The alternate hypothesis is always one of these:

$$H_1: \text{median} < (\text{stated median}) \quad H_1: (\text{After}) - (\text{Before}) < 0$$

$$H_1: \text{median} = (\text{stated median}) \quad H_1: (\text{After}) - (\text{Before}) = 0$$

$$H_1: \text{median} > (\text{stated median}) \quad H_1: (\text{After}) - (\text{Before}) > 0$$

Test statistic:

When $n < 26$, $n = \text{sum of } x^+ \text{ and } x^-$

smaller of:

x^+ , the # of data points greater than median (or 0)

and

x^- , the # of data points less than the median (or 0)

or **When $n > 25$, $n = \text{sum of } x^+ \text{ and } x^-$**

$$z_{\text{sign}} = \frac{x^+ - n/2}{\sqrt{n}/2}$$

$$n = x^+ + x^-, \quad n > 25$$

binomial distribution used where $p=.5$,

$n = \text{the sum of } x^+ \text{ and } x^-$.

The z table is used with x in formula as the smaller x^+ and x^- .

One page [Summary](#) , sample problem, and a [spreadsheet](#).

Wilcoxon Rank Sum Test

This tests to see if evidence exists that 2 distributions are the same by virtue of their symmetry, center, and the distribution of ranks (with averaged ranks awarded to tied data points). Do two populations have the same distribution, are their centers and spreads the same?

Needed:

2 not necessarily normal distributions

each sample size is 10 or greater

independent random samples

The null hypothesis is always:

H_0 : The distributions are the same.

(The means and rank sums are "the same.")

Some alternate hypothesis are:

H_1 : The distributions are not the same.

Are the means and rank sums close but not "the same?" (\neq)

H_1 : The first distributions is to the left of the other.

Is the first's rank sums much smaller, data points more to the left, than the second's? ($<$)

H_1 : The first distributions is to the right of the same.

Is the first's rank sums much larger, data points more to the right, than the second's? ($>$)

Test statistic: **The sample size determines the labels in the formulas.**

The label n_1 is given to the smaller sample size -- fewer numbers to add.

The data is then POOLED, with averaged ranks awarded to tied data points.

Individual r_1 and r_2 are computed by adding the ranks of the data points from the samples.

R is r_1 , the sum of the ranks of the sample with the smaller sample size.

$$z_{Wilcoxon_{rank\ sum}} = \frac{R - \mu_R}{\sigma_R}$$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

One page [Summary](#) , sample problem, and a [spreadsheet](#).

Wilcoxon Signed-Rank Test

This tests to see if evidence exists that the medians of two distributions are the same, or before-after differences exist.

Needed:

- n samples paired as $x_1 - x_2 = \text{difference} \neq 0$.
- the ranks of the pooled absolute values of the differences, tie ranks averaged.
- "signed-rank" products = (original sign)(pooled rank)
- the absolute value of the sum of the products of the (original sign)(pooled rank) of the negative differences.
- the absolute value of the sum of the products of the (original sign)(pooled rank) of the positive differences.
- the SMALLER of the absolute values of the sums of the + or - signed-rank products
- 2 not necessarily normal distributions
- independent random samples

The null hypothesis is always:

$$H_0: \text{median}_1 = \text{median}_2, \text{ or } H_0: \text{before} - \text{after} = 0$$

Some alternate hypothesis are:

$$H_1: \text{median}_1 \neq \text{median}_2, \text{ or } H_1: \text{before} - \text{after} \neq 0$$

$$H_1: \text{median}_1 > \text{median}_2, \text{ or } H_1: \text{before} - \text{after} > 0$$

$$H_1: \text{median}_1 < \text{median}_2, \text{ or } H_1: \text{before} - \text{after} < 0$$

Test statistic:

if $n < 30$, use table for critical value and the SMALLER for the test statistic

if $n \geq 30$, use z distribution for critical value and formula

with $\mu = n(n+1)/4$ for test statistic OR

if $n \geq 30$, use z distribution for critical value and

the absolute value of the sum of all (sign)(rank) in formula

with $\mu = .5$ for test statistic

$$z = \frac{W - 0.5}{\sqrt{\frac{n(n+1)(2n+1)}{6}}}$$

$$W = \left| \sum [\text{sgn}(x_2 - x_1) \cdot R] \right|$$

$$z_{Wilcoxon_{signed-rank}} = \frac{w_s - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$



One page [Summary](#) , sample problem, and a [spreadsheet](#).

Spearman rank correlation coefficient

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Kruskal-Wallis test

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \dots + \frac{R_k^2}{n_k} \right) - 3(N+1)$$

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www.mathnstuff.com/math/spoken/here/2class/90/htest2.htm