Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \csc u = \frac{1}{\sin u}$$

$$\cos u = \frac{1}{\sec u} \qquad \sec u = \frac{1}{\cos u}$$

$$\tan u = \frac{1}{\cot u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u}$$

$$\cot u = \frac{\cos u}{\sin u}$$

Cofunctions Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \csc(-u) = -\csc u$$

$$\cos(-u) = \cos u \qquad \sec(-u) = \sec u$$

$$\tan(-u) = -\tan u \qquad \cot(-u) = -\cot u$$

Sum and Difference formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Veronica's Bow Tie & All Students Try Cheating

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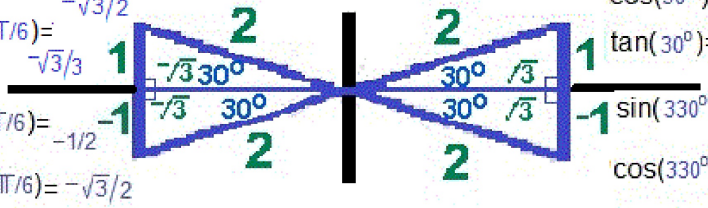
1. In each quadrant, enter the degree and radian measure of the hypotenue angle.
2. Evaluate each trig function. (See quadrant II of the 30s.)
3. Explain the 2 title expressions.

The 30s



$\sin(150^\circ) = \sin(5\pi/6) = 1/2$

$\cos(150^\circ) = \cos(5\pi/6) = -\sqrt{3}/2$

$\tan(150^\circ) = \tan(5\pi/6) = -\sqrt{3}/3$

$\sin(210^\circ) = \sin(7\pi/6) = -1/2$

$\cos(210^\circ) = \cos(7\pi/6) = -\sqrt{3}/2$

$\tan(210^\circ) = \tan(7\pi/6) = \sqrt{3}/3$

$\sin(30^\circ) = \sin(\pi/6) = 1/2$

$\cos(30^\circ) = \cos(\pi/6) = \sqrt{3}/2$

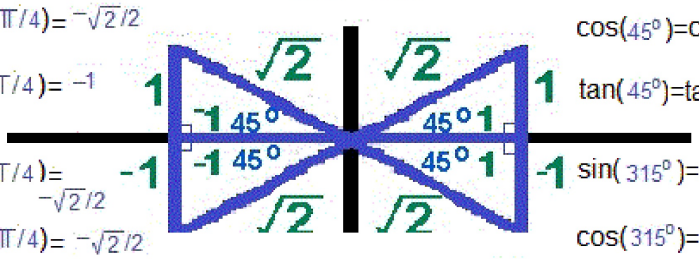
$\tan(30^\circ) = \tan(\pi/6) = \sqrt{3}/3$

$\sin(330^\circ) = \sin(11\pi/6) = -1/2$

$\cos(330^\circ) = \cos(11\pi/6) = \sqrt{3}/2$

$\tan(330^\circ) = \tan(11\pi/6) = -\sqrt{3}/3$

The 45s



$\sin(135^\circ) = \sin(3\pi/4) = \sqrt{2}/2$

$\cos(135^\circ) = \cos(3\pi/4) = -\sqrt{2}/2$

$\tan(135^\circ) = \tan(3\pi/4) = -1$

$\sin(225^\circ) = \sin(5\pi/4) = -\sqrt{2}/2$

$\cos(225^\circ) = \cos(5\pi/4) = -\sqrt{2}/2$

$\tan(225^\circ) = \tan(5\pi/4) = 1$

$\sin(45^\circ) = \sin(\pi/4) = \sqrt{2}/2$

$\cos(45^\circ) = \cos(\pi/4) = \sqrt{2}/2$

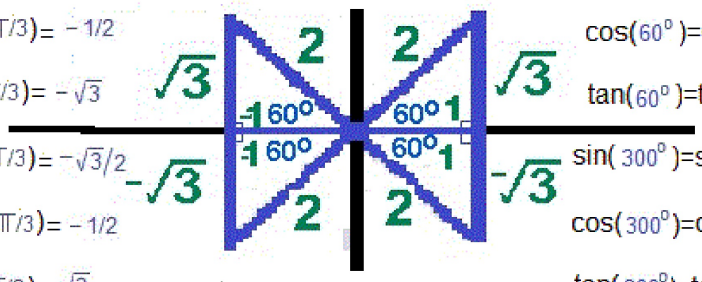
$\tan(45^\circ) = \tan(\pi/4) = 1$

$\sin(315^\circ) = \sin(7\pi/4) = -\sqrt{2}/2$

$\cos(315^\circ) = \cos(7\pi/4) = \sqrt{2}/2$

$\tan(315^\circ) = \tan(7\pi/4) = -1$

The 60s



$\sin(120^\circ) = \sin(2\pi/3) = \sqrt{3}/2$

$\cos(120^\circ) = \cos(2\pi/3) = -1/2$

$\tan(120^\circ) = \tan(2\pi/3) = -\sqrt{3}$

$\sin(240^\circ) = \sin(4\pi/3) = -\sqrt{3}/2$

$\cos(240^\circ) = \cos(4\pi/3) = -1/2$

$\tan(240^\circ) = \tan(4\pi/3) = \sqrt{3}$

$\sin(60^\circ) = \sin(\pi/3) = \sqrt{3}/2$

$\cos(60^\circ) = \cos(\pi/3) = 1/2$

$\tan(60^\circ) = \tan(\pi/3) = \sqrt{3}$

$\sin(300^\circ) = \sin(5\pi/3) = -\sqrt{3}/2$

$\cos(300^\circ) = \cos(5\pi/3) = 1/2$

$\tan(300^\circ) = \tan(5\pi/3) = -\sqrt{3}$

Double-Angle formulas

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$

Power-Reducing formulas

$\sin^2 u = \frac{1 - \cos 2u}{2}$ $\cos^2 u = \frac{1 + \cos 2u}{2}$ $\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$

Half-Angle formulas

$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$

$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$

$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$

The signs of $\sin(u/2)$ and $\cos(u/2)$ depend on the quadrant in which $u/2$ lies.

Product-to-Sum formulas

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

Sum-to-Product formulas

$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$

$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$

$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$

$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$