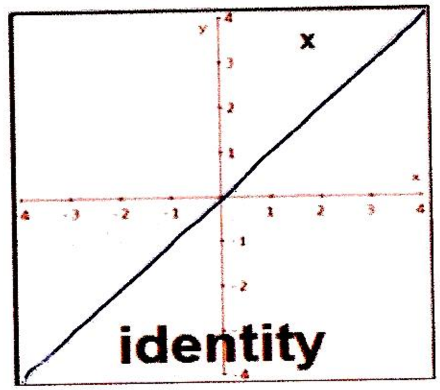
Functions Learned in Precalc Which are Needed in Calc

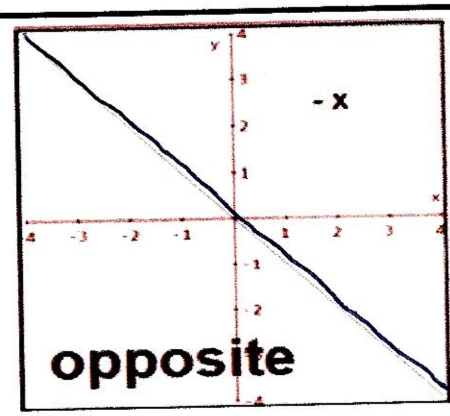
identity function	x
the opposite function	$-x$
the reciprocal function	$1/x$ or x^{-1}
a constant function	c, c = some constant
the "twice" function	$2x$
the squaring function	x^2
the square root function	√(x), or $x^{1/2}$
the absolute value function	$\|x\|$
the greatert integer function	[[x]]
piece-wise defined functions	
a polynomial function	a polynomial function
a rational function	a rational function
an exponential or power function	c^x where $c > 0$ and $c \neq 1$
the exponential function	exp(x) or e^x
a lograthmic function	$\log_c(x)$, $c > 0$
the natual log function	ln(x) or $\log_e(x)$
sine function	sin(x)
cosine function	cos(x)
tangent function	tan(x)
cosecant function	csc(x)
secant function	sec(x)
cotangent function	cot(x)
Arcsine function	$\sin^{-1}(x)$
Arccosine function	$\cos^{-1}(x)$
Arctangent function	$\tan^{-1}(x)$
hyperbolic sine function	sinh(x)
hyperboic cosine function	cosh(x)
hyperbolic tangent function	tanh(x)

function in words	the identity function
f(x) in symbols	x
domain	all reals (- ∞, + ∞)
range	all reals (- ∞, + ∞)
features	each y is identically equal to its x
period	none
x-intercept(s)	(0,0)
y-intercept(s)	(0,0)
reciprocal function	x, itself
inverse function	x, itself
asymptote(s), discontinuities	none
continuous?	yes
derivative	1
anti-derivative	$x^2 / 2 + c$



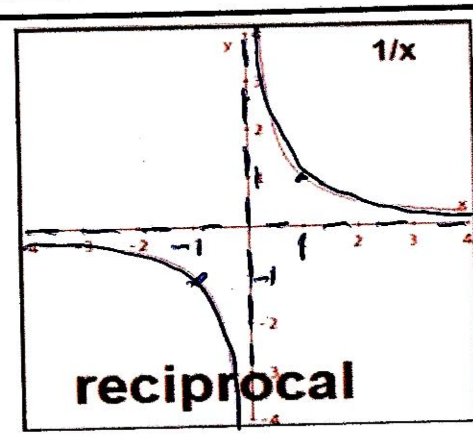
* to "undo," take the inverse, use the original number, x
* makes a 45^{o} angle w/the positive x-axis

function in words	the opposite function
f(x) in symbols	- x
domain	all reals (- ∞, + ∞)
range	all reals (- ∞, + ∞)
features	each y is the opposite of its x
period	none
x-intercept(s)	(0,0)
y-intercept(s)	(0,0)
reciprocal function	- 1/x
inverse function	-x, itself
asymptote(s), discontinuities	none
continuous?	yes
derivative	- 1
anti-derivative	$- x^2 / 2 + c$



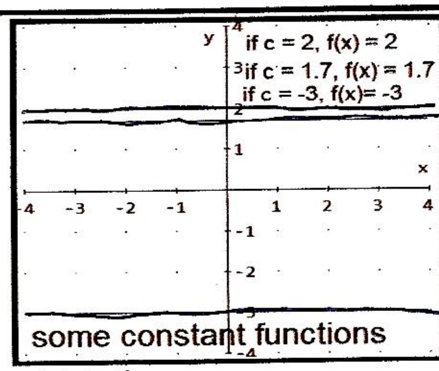
* to "undo," take the inverse, use the opposite of what you have, -x
* makes a 45^{o} angle w/the positive negative x-axis

function in words	the reciprocal function
f(x) in symbols	1/x or x^{-1}
domain	all reals except 0, $x \neq 0$
range	all reals except 0, $y \neq 0$
features	a hyperbola w/2 branches
period	none
x-intercept(s)	none
y-intercept(s)	none
reciprocal function	1/x, itself
inverse function	1/x, itself
asymptote(s), discontinuities	vertical asymptote @ x=0
	horizontal asymptote at y= 0
continuous?	no
derivative	1
anti-derivative	ln|x| + c

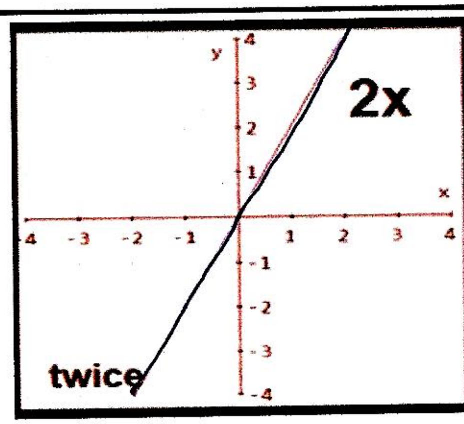


* the cosecant, secant, and cotangent are reciprocals of the sine, cosine, and tangent

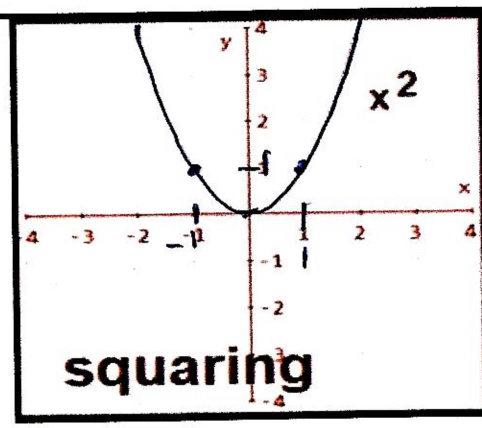
function in words	a constant function
f(x) in symbols	c, c = some constant
domain	all reals (- ∞, + ∞)
range	c, whatever constant is used
features	horizontal line
period	none
x-intercept(s)	only if c = 0 then every point
y-intercept(s)	(0, c)
reciprocal function	1/c
inverse function	none, doesn't pass horizontal line test, not 1-to-1
asymptote(s), discontinuities	no
continuous?	yes, it is a polynomial
derivative	0
anti-derivative	cx + d, where d is some constant



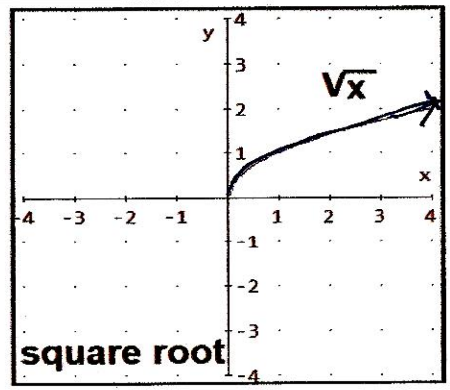
function in words	the "twice" function
f(x) in symbols	2x
domain	all reals (- ∞, + ∞)
range	all reals (- ∞, + ∞)
features	line w/a slope of 2
period	none
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	x/2
inverse function	
asymptote(s), discontinuities	
period	
continuous?	
derivative	2
anti-derivative	x3 /3 + c



function in words	the squaring function
f(x) in symbols	x^2
domain	all reals (- ∞, + ∞)
range	y ≥ 0, [0. + ∞)
features	U-shaped, vertex at (0, 0)
period	none
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	$1/x^2$
inverse function	restricted domain, square root function
asymptote(s), discontinuities	none
continuous?	yes
derivative	2x
anti-derivative	$2x^3 /3 + c$



function in words	the square root function
f(x) in symbols	√(x), or $x^{1/2}$
domain	*See note [0, ∞), x≥ 0
range	y ≥ 0, [0. + ∞)
features	looks like half a parabola on its side
period	none
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	1/√(x) is √(x)/x is also $x^{-1/2}$
inverse function	x^2
asymptote(s), discontinuities	none
continuous?	yes
derivative	1/(2√(x)) or x^(-1/2)/2
anti-derivative	(2/3)x^(3/2) + c



* The square root of negative numbers are perfectly good numbers, but, here graphing is on the real plane and these are complex numbers.

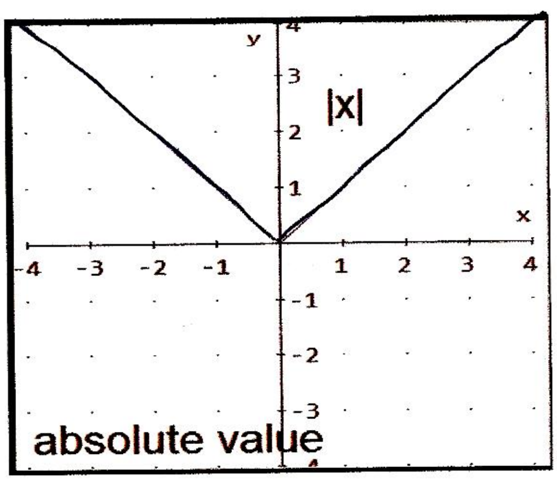
function in words	the absolute value function
f(x) in symbols	|x| *See note
domain	all reals (- ∞, + ∞)
range	y ≥ 0, [0. + ∞)
features	V-shaped, vertex at (0, 0)
period	none
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	1/||x|
inverse function	none, doesn't pass horizontal line test, not 1-to-1
asymptote(s), discontinuities	no
continuous?	yes
derivative	see below
anti-derivative	see below

* This function is really √(x²)

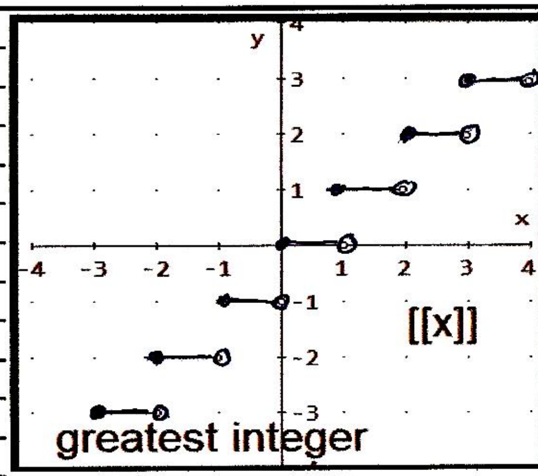
* See piece-wise below

$$y = |x| \qquad \frac{dy}{dx} = \begin{cases} -1, x < 0 \\ 0,\ x = 0 \\ 1, x > 0 \end{cases}$$

$$\int |x|\, dx = \begin{cases} -1x^2/2 + c, x < 0 \\ x\ , x = 0 \\ x^2/2 + c, x > 0 \end{cases}$$



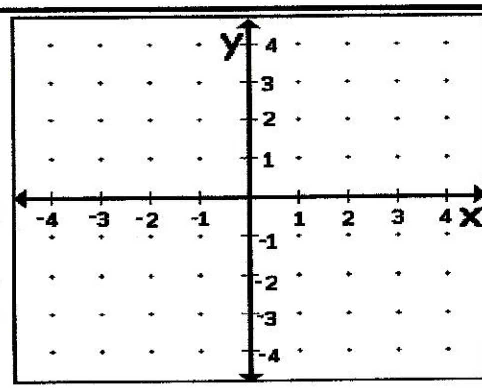
function in words	the greatest integer function
f(x) in symbols	[[x]]
domain	all reals (- ∞, + ∞)
range	integers {…,-3,-2,-1,0,1,2,3,…}
features	looks like steps
period	none
x-intercept(s)	each point 0 ≤ x < 1
y-intercept(s)	(0, 0)
inverse function	none, doesn't pass horizontal line test, not 1-to-1
asymptote(s), discontinuities	discontinuous, no asymptotes
continuous?	no
derivative	see at right
anti-derivative	see at right



$y = [[x]]$, for each interval [a,b)

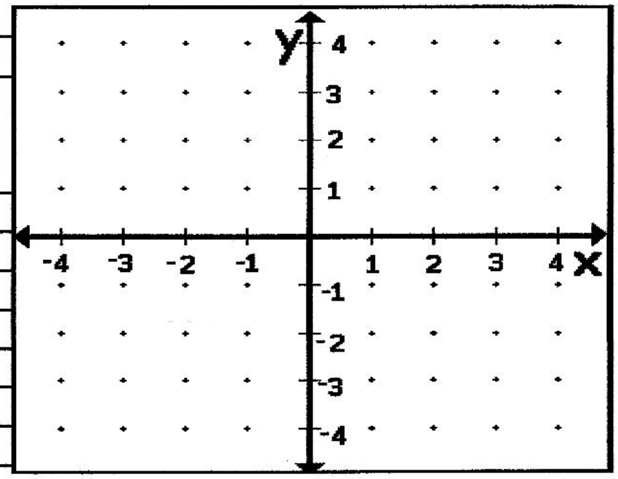
$\frac{dy}{dx} = a \quad \int [[x]]dx = x^2 + c$

function in words	a piece-wise defined function
f(x) in symbols	varies
domain	varies
range	varies
features	may be broken
period	maybe
x-intercept(s)	varies
y-intercept(s)	varies
reciprocal function	varies
inverse function	varies
asymptote(s), discontinuities	maybe
continuous?	maybe
derivative	varies
anti-derivative	varies



* the absolute value may be defined
as a piece-wise defined function
as a piece-wise defined function
f(x) = - x, for x < 0
f(x) = x for x ≥ 0

function in words	a polynomial function
f(x) in symbols	$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0 x^0$
domain	all reals $(-\infty, +\infty)$
range	all reals $(-\infty, +\infty)$
features	
features	
period	none
x-intercept(s)	at least one, probably many
y-intercept(s)	maybe one
reciprocal function	a rational function
inverse function	maybe
asymptote(s), discontinuities	no
continuous?	yes
derivative	see below
anti-derivative	see below

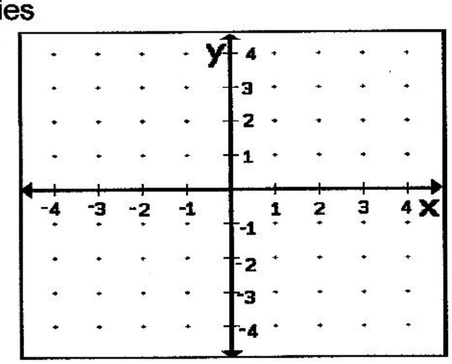


$$\int y\,dx = \frac{a_n x^{n+1}}{(n+1)} + \frac{a_{n-1} x^n}{(n)} + \frac{a_{n-2} x^{n-1}}{(n-1)} + \ldots + \frac{a_3 x^4}{(4)} + \frac{a_2 x^3}{(3)} + \frac{a_1 x^2}{(2)} + \frac{a_0 x^1}{(1)} + c$$

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0 x^0$$

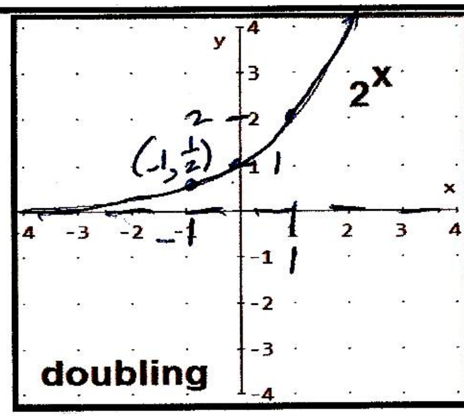
$$y' = a_n (n) x^{n-1} + a_{n-1}(n-1) x^{n-2} + a_{n-2}(n-2) x^{n-3} + \ldots + a_3 (3) x^2 + a_2 (2) x^1 + a_1 (1) x^0$$

function in words	a rational function
f(x) in symbols	$\dfrac{a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0 x^0}{b_n x^n + b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \ldots + b_2 x^2 + b_1 x^1 + b_0 x^0}$
domain	varies, check denominator function
range	varies
features	probably asymptotes, maybe discontinuities
x-intercept(s)	maybe
y-intercept(s)	maybe
reciprocal function	maybe a polynomial function
inverse function	not necessarily
asymptote(s), discontinuities	yes, maybe
period	not necessarily
continuous?	not likely
derivative	see below. u(x) & v(x) are polynomials
anti-derivative	varies



$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{u'(x) \bullet v(x) - v'(x) \bullet u(x)}{[v(x)]^2}$$

function in words	an exponential or power function
f(x) in symbols	b^x where $b > 0$ and $b \neq 1$
domain	all reals $(-\infty, +\infty)$
range	$y > 0$, $(0. +\infty)$
features:	growth, reciprocal is decay, the halving function
period	no
x-intercept(s)	no
y-intercept(s)	(0, 1)
reciprocal function	b^{-x}
inverse function	$\log_b(x)$
asymptote(s), discontinuities	$y = 0$
continuous?	yes
derivative	$(b^x)\ln(b)$
anti-derivative	b^x/ln(b)+ c

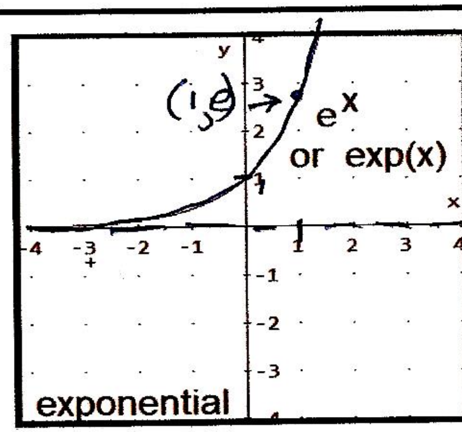


$f(x) = 2^x$

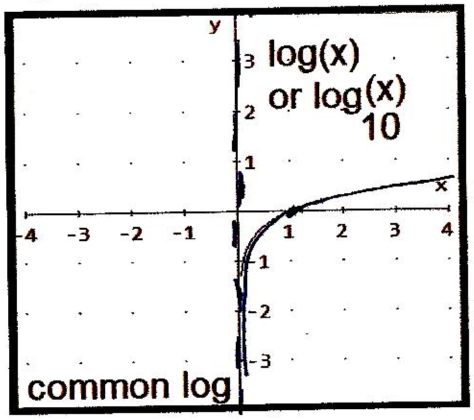
reciprocal $f(x) = 1/f(x) =$

$1/2^x = (1/2)^x = (2^{-1})^x = 2^{-x}$

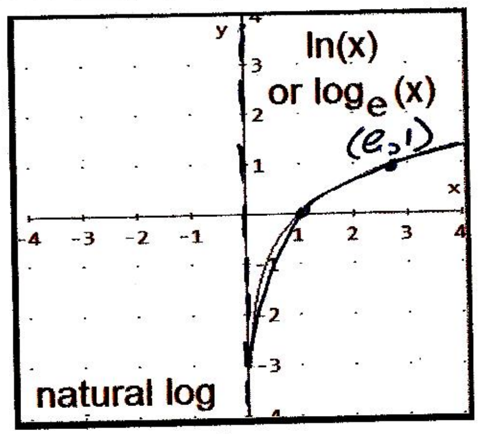
function in words	the exponential function
f(x) in symbols	exp(x) or e^x
domain	all reals $(-\infty, +\infty)$
range	$y > 0$, $(0. +\infty)$
features	used for growth and decay
period	no
x-intercept(s)	no
y-intercept(s)	(0, 1)
reciprocal function	e^{-x}
inverse function	$\log_e(x) = \ln(x)$
asymptote(s), discontinuities	$y = 0$
continuous?	yes
derivative	e^x
anti-derivative	$e^x + c$



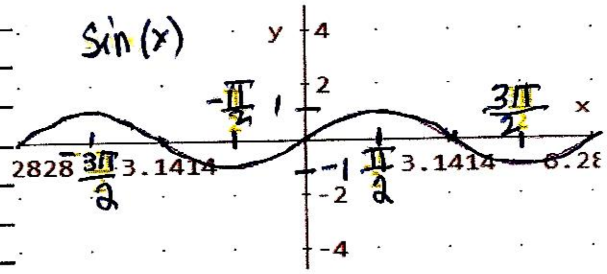
function in words	a lograthmic function
f(x) in symbols	$\log_b(x)$, $b > 0$, here $b = 10$
domain	$x > 0$, $(0, \infty)$
range	all reals $(-\infty, +\infty)$
features	
period	no
x-intercept(s)	(1, 0)
y-intercept(s)	none
reciprocal function	$1/\log_b(x)$
inverse function	b^x
asymptote(s), discontinuities	$x = 0$, no discontinuities
continuous?	yes
derivative	$1/[x\ln(b)]$
anti-derivative	$[x(\ln(x) - 1)]/\ln(b) + c$



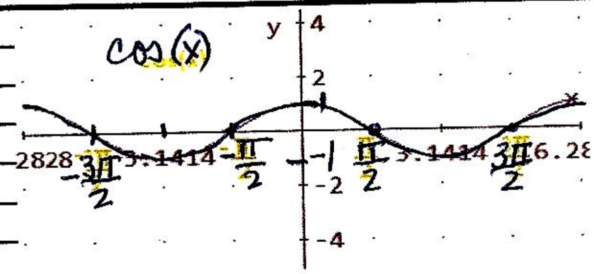
function in words	the natual log function
f(x) in symbols	$\ln(x)$ or $\log_e(x)$
domain	$x > 0$, $(0, \infty)$
range	all reals $(-\infty, +\infty)$
features	
period	none
x-intercept(s)	(1, 0)
y-intercept(s)	none
reciprocal function	$1/\ln(x)$
inverse function	e^x
asymptote(s), discontinuities	no discontinuities, $y = 0$
continuous?	yes
derivative	$1/x$
anti-derivative	$x(\ln(x) - 1) + c$



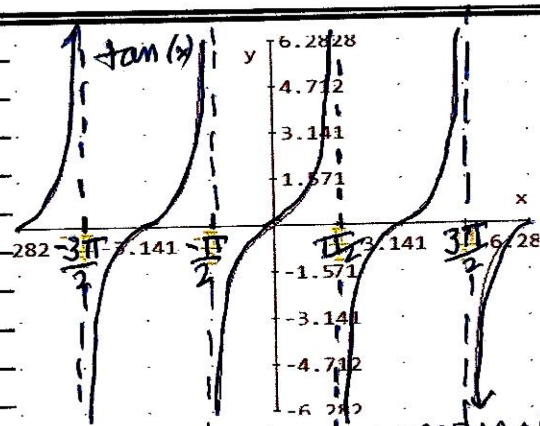
function in words	sine function
f(x) in symbols	sin(x)
domain	all reals $(-\infty, +\infty)$
range	$-1 \leq x \leq 1$
features	it is the cosine function shifted
period	2π
x-intercept(s)	$0 \pm n\pi$
y-intercept(s)	(0, 0)
reciprocal function	cosecant, scs(x)
inverse function	Arcsine(x), $\sin^{-1}(x)$
asymptote(s), discontinuities	no
continuous?	yes
derivative	cos(x)
anti-derivative	-cos(x) + c



function in words	cosine function
f(x) in symbols	cos(x)
domain	all reals $(-\infty, +\infty)$
range	$-1 \leq x \leq 1$
features	it is the sine function shifted
period	2π
x-intercept(s)	$\pi/2 \pm n\pi$
y-intercept(s)	(1, 0)
reciprocal function	secant, sec(x)
inverse function	Arccosine, $\cos^{-1}(x)$
asymptote(s), discontinuities	no
continuous?	yes
derivative	- sin(x)
anti-derivative	sin(x) + c



function in words	tangent function
f(x) in symbols	tan(x)
domain	all reals except $x \neq \pi/2 \pm n\pi$
range	all reals $(-\infty, +\infty)$
features	vertical asymptotes - see domain
period	π
x-intercept(s)	(0,0) and $(0 \neq n\pi, 0)$
y-intercept(s)	(0,0)
reciprocal function	cotangent f(x), cot(x)
inverse function	Arctangent f(x)
asymptote(s), discontinuities	vertical asymptotes - see domain
continuous?	no
derivative	$\sec^2(x)$
anti-derivative	ln(sec(x)) + c

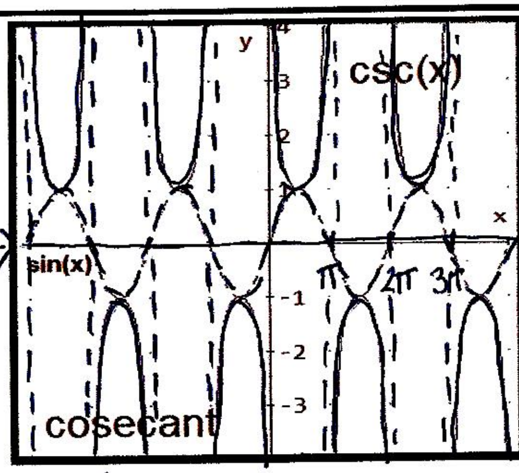


$\tan(x) = \frac{\sin(x)}{\cos(x)}$ ← zeros of sin(x) are zeros of tan(x)

→ zeros of cosine are asymptotes of tan(x)

function in words	cosecant function
f(x) in symbols	csc(x)
domain	x ≠ 0 ± nπ
range	y ≥ 1 and y ≤ -1
features	undefined when sine is 0
period	2π
x-intercept(s)	none
y-intercept(s)	none
reciprocal function	sine, sin(x)
inverse function	Arccosecant
asymptote(s), discontinuities	x=0, x= 0 ± nπ
period	2π
continuous?	none
derivative	-csc(x)cot(x)
anti-derivative	ln|csc(x)-cot(x)|+ c

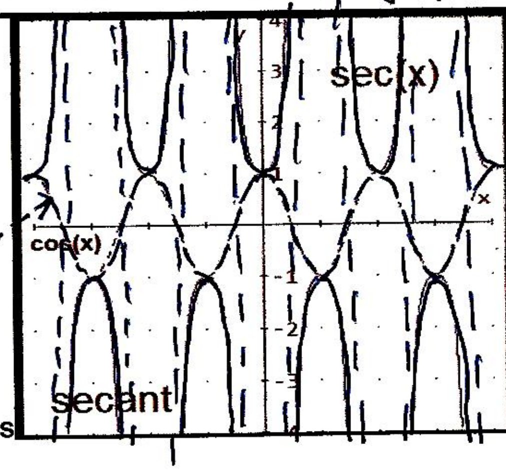
sin(x)



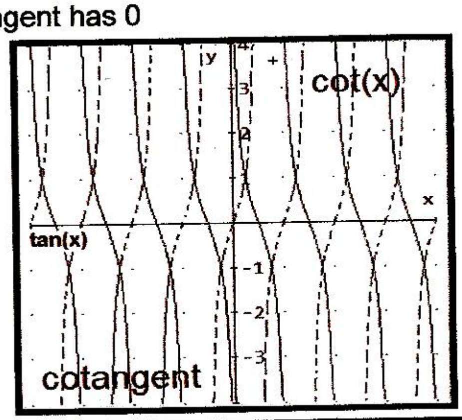
when reciprocal has a zero, function has an asymptote

function in words	secant function
f(x) in symbols	sec(x)
domain	x ≠ π/2 ± nπ
range	y ≥ 1 and y ≤ -1
features	undefined when cosine is 0
period	2π
x-intercept(s)	none
y-intercept(s)	(0, 1)
reciprocal function	cosine
inverse function	Arcsecant
asymptote(s), discontinuities	no discontinuities, vertical asymptotes
continuous?	no
derivative	sec(x)tan(x)
anti-derivative	ln|sec(x) + tan(x)| + c

cos(x)



function in words	cotangent function
f(x) in symbols	cot(x)
domain	x ≠ 0 ± nπ
range	all reals
features	whenever tangent is undefined, cotangent has 0
period	π
x-intercept(s)	(π/2, 0), (π/2 ± nπ, 0)
y-intercept(s)	none
reciprocal function	tangent
inverse function	Arccotangent
asymptote(s), discontinuities	x = π/2, x = π/2 ± nπ
continuous?	no
derivative	- csc(x)csc(x)
anti-derivative	ln|sin(x)| + c

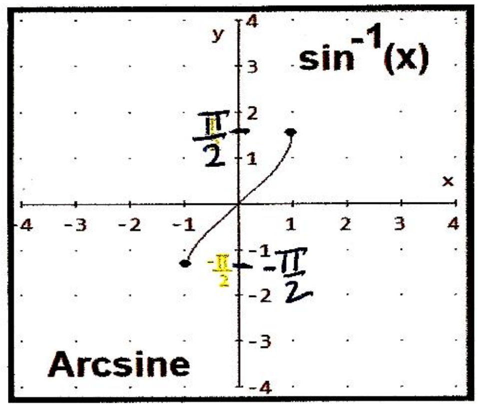


function in words	Arcsine function
f(x) in symbols	$\sin^{-1}(x)$
domain	$-1 \le x \le 1$
range	$-\pi/2 \le x \le \pi/2$
features	
period	none
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	$1/\sin^{-1}(x)$
inverse function	sin(x) with domain restriction
asymptote(s), discontinuities	none
continuous?	yes
derivative	
anti-derivative	

$$\int y\,dx = x\arcsin(x) + \sqrt{1-x^2} + c$$

$$y = \arcsin(x)$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

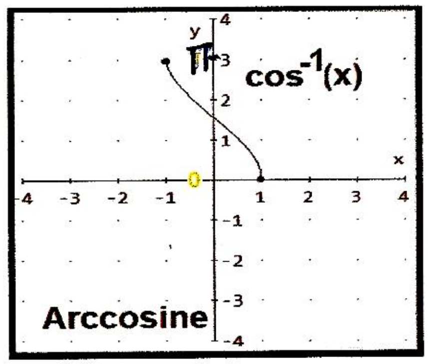


function in words	Arccosine function
f(x) in symbols	$\cos^{-1}(x)$
domain	$-1 \le x \le 1$
range	$0 \le x \le \pi$
features	
period	none
x-intercept(s)	(1, 0)
y-intercept(s)	(0, π/2)
reciprocal function	1/Arccos(x)
inverse function	cos(x) w/restricted domain
asymptote(s), discontinuities	none
continuous?	yes
derivative	
anti-derivative	

$$\int y\,dx = x\arccos(x) - \sqrt{1-x^2} + c$$

$$y = \arccos(x)$$

$$y' = \frac{-1}{\sqrt{1-x^2}}$$

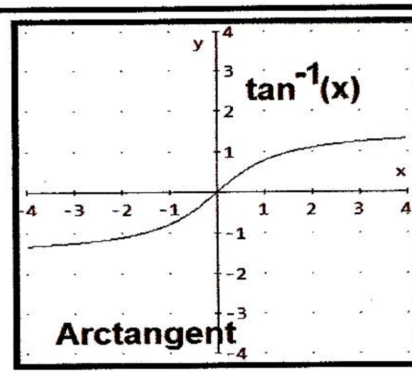


function in words	Arctangent function
f(x) in symbols	$\tan^{-1}(x)$
domain	all reals
range	(-1, 1)
features	
period	no
x-intercept(s)	(0, 0)
y-intercept(s)	(0, 0)
reciprocal function	1/Arctan(x)
inverse function	tan(x) for (-π/2, π/2)
asymptote(s), discontinuities	y = 1, y= -1
continuous?	yes
derivative	
anti-derivative	

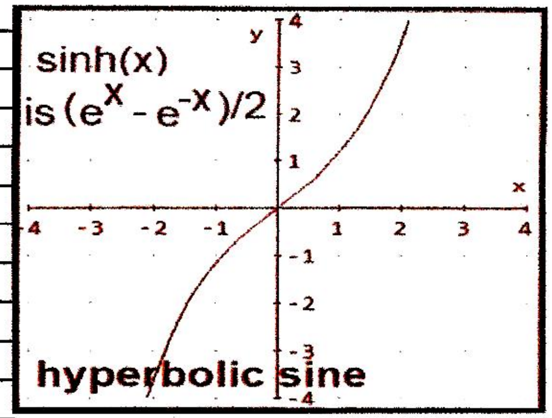
$$\int y\,dx = x\arctan(x) - \ln\sqrt{1+x^2} + c$$

$$y = arc\tan(x)$$

$$y' = \frac{1}{1+x^2}$$

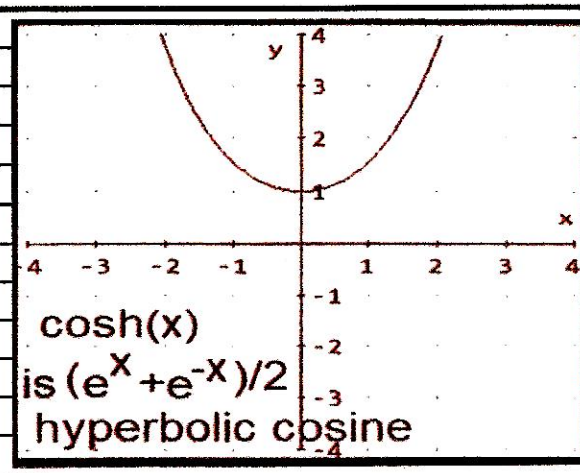


function in words	hyperbolic sine function
f(x) in symbols	sinh(x)
domain	all reals
range	all reals
features	
period	none
x-intercept(s)	(0,0)
y-intercept(s)	(0,0)
reciprocal function	csch(x)
inverse function	
asymptote(s), discontinuities	none
period	no
continuous?	
derivative	
anti-derivative	



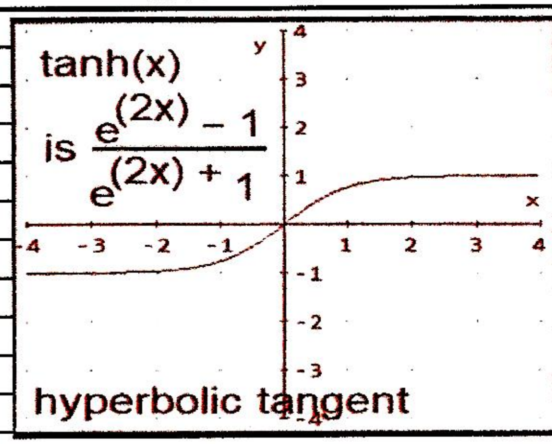
SEE BELOW.

function in words	hyperbolic cosine function
f(x) in symbols	cosh(x)
domain	all reals
range	[1, ∞)
features	
period	no
x-intercept(s)	none
y-intercept(s)	(0, 1)
reciprocal function	sech(x)
inverse function	
asymptote(s), discontinuities	
period	
continuous?	
derivative	
anti-derivative	



SEE BELOW.

function in words	hyperbolic tangent function
f(x) in symbols	tanh(x)
domain	
range	
features	
period	
x-intercept(s)	
y-intercept(s)	
reciprocal function	
inverse function	
asymptote(s), discontinuities	
continuous?	
derivative	
anti-derivative	



SEE BELOW.

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})) \ \ \forall\ x$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})) \ \ \forall\ x \geq 1$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \tanh^{-1}(x) = \frac{1}{2}[\ln(\frac{1+x}{1-x}) \ \ \forall\ |x| < 1$$

$$\operatorname{csch}(x) = \frac{2}{e^x - e^{-x}} \qquad \operatorname{csch}^{-1}(x) = \ln(x + \sqrt{(x^2 - 1)}))$$

$$\operatorname{sech}(x) = \frac{2}{e^x + e^{-x}} \qquad \operatorname{sech}^{-1}(x) = \ln(\frac{1 + \sqrt{(1 - x^2)}}{x}) \ \ \forall\ 0 < x \leq 1$$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \qquad \coth^{-1}(x) = \frac{1}{2}\ln(\frac{x+1}{x-1}) \ \ \forall\ |x| > 1$$