



Slope in Sound & Picture

Page TOC, Video, & pdf

Before studying slope, one needs to understand graphing and the coordinate plane. See [A Graph is A Portrait](#). If you understand these ideas continue.

Here are the [video](#) and [pdf](#), and [Geometer's Sketchpad](#) of/for this page.

The page approaches slope in a few ways. These are: VERBAL / Auditory, WRITTEN / Symbolic, PICTORIAL / Visual, CONCRETE / Kinesthetic.

VERBAL / Auditory

- [History & An Urban Legend](#) ⇒
- [A Slope of Zero](#) ⇒
- [A Slope of One](#) ⇒
- [Slope -- How Are Things Going? A Measure of Change](#) ⇒

PICTORIAL / Visual

- [The Scale Called Slope](#) ⇒

CONCRETE / Kinesthetic.

- [The Slopemeter](#) ⇒

WRITTEN / Symbolic

- [Slope As Numbers](#) ⇒
- [Slope in Sounds](#) ⇒
- [Rise Over Run with Coordinates](#) ⇒
- [Geometer's Sketchpad, sheets](#) ⇒

Aside ⇒

History & An Urban Legend

[Renee Descartes and Pierre Fermat](#) (see the [Slope Song Sketchpad](#), sheet 0) are credited with the creation of the coordinate, Cartesian, plane in 1635. Each of the two stories related here involve Renee Descartes. The first is fact. The second is urban legend, believed true but with no documentation or proof as to its validity.

As a child, Descartes was sickly an avid scholar and bed-ridden, before becoming a soldier, philosopher ("I think, therefore I am"), mathematician, and teacher. History says he created the coordinate plane for tracking a fly as it crawled about the tiled ceiling above his bed.

The urban legend relates the use of the letter, symbol, "m" for [slope](#).

Often when one sees a mountain, one thinks of the slope of the mountain. Does the mountain slope up? Does the mountain slope down? Is there a mountain or just a flat plain? Slope has nothing to do with lines or curves but does describe the tilt of a mountain or land.

Think of that smart French kid laying in bed then looking out the window at mountains, valleys, plains.

He who coordinatized the ceiling might well have coordinatized the world outside his window, the mountains, the valleys, the plains. The first letter of mountain in French as it is in English is "m." Think "m" for the slope of a mountain.

A mountain slope which goes up from a point would be described as a positive slope. A mountain which has a lower altitude as one walks would have a negative slope because the opposite of up is down and the opposite of positive is negative.

Think of a flat plain as having no incline. Zero is a logical choice for the slope of land on which one doesn't walk down or doesn't walk up. Between negative and positive in numbers is zero.

The "m" for slope story is just an urban legend but the negative for down, zero for flat, and positive for up makes sense for slope. Next see how it might work with sound.

A Slope of Zero

[Listen to a slope of zero.](#)

Does this sound flat, like a flat plain? The notes/tones/frequencies do not increase (go up) or decrease (go down). It's the same tone repeated.

[Listen to a slope of one.](#) Then, listen again. The pitch goes higher for each unit of time.

We'll use this as the [unit](#) of measure of slope. We'll judge other slopes in comparison to this.

A free **Geometer's Sketchpad!** may be downloaded from this link, so the reader can experiment with a slope of 0, seeking where a function is flat or has other values.

A number of sheets of [ssong.gsp](#) are provided so the reader might become the experimenter and examine slope in the graphs provided.

The [video](#) might help the reader experiment with functions such as a constant function, a line, a cubic (really good for checking out the slope of hills and finding the flat plain where the slope is zero. Sheets are also included to check out the polynomial and rational functions.

One might experiment to figure out the meaning on "no slope" or "undefined" for a vertical line.

The Scale Called Slope

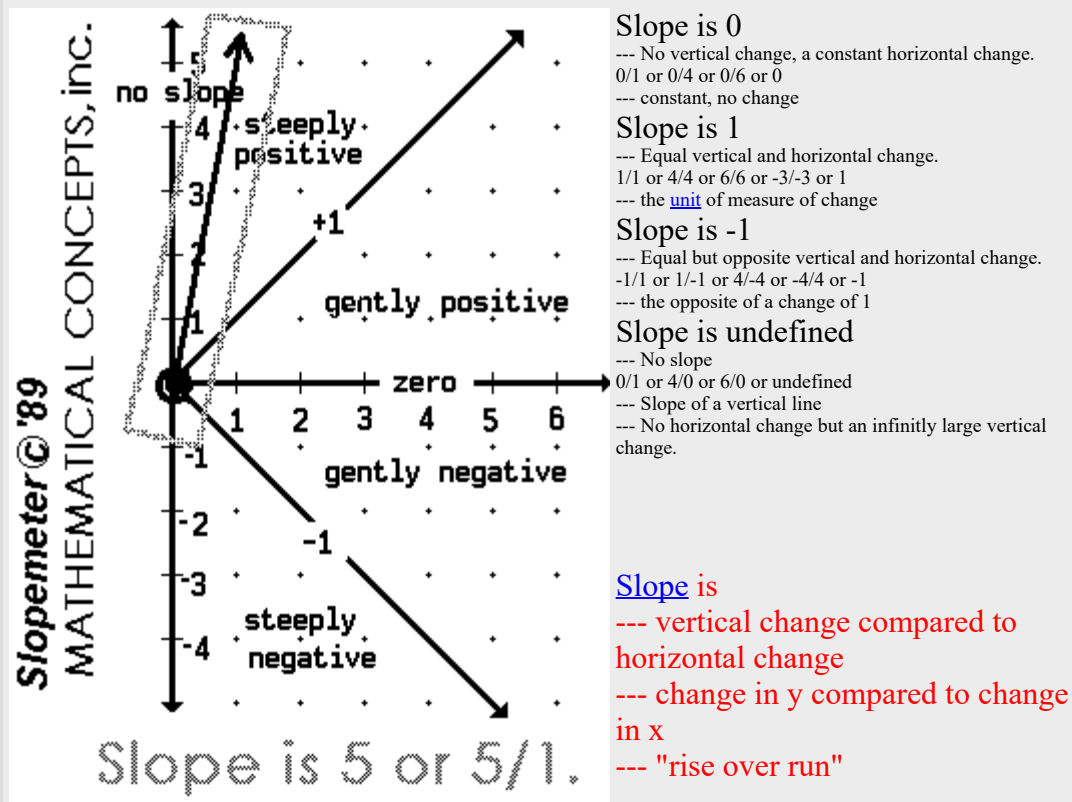
You've listened to a slope of one and you've listened to a slope of zero. Using two measurements, a scale or gauge is possible. Two measurements are required -- the zero and the one. With these established, other measurements follow.

Listen to a [slope of zero](#) and to a [slope of one](#).

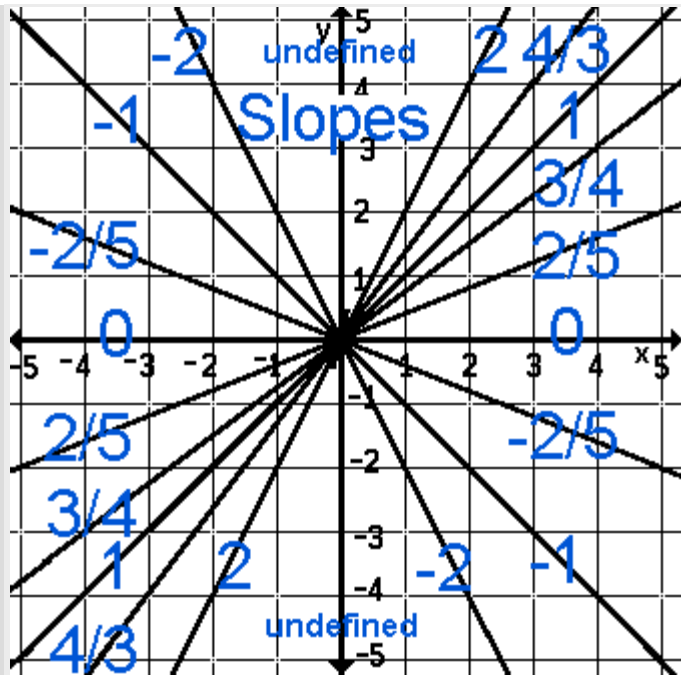
- Do the sounds make sense to you?
- What is the same in [0](#) and [1](#).
- What is different in [0](#) and [1](#).
- What should -1 sound like?
- What should 2 sound like?

Before you scroll down on this page, press RELOAD or REFRESH on your browser so the next graphic has animation, then, just sit there, examine the graphic, and think about slope.

The Slopemeter



Slope in Sounds



Experiment.

Predict the sound of each slope before clicking on the line with the desired slope.

Again **Slope** is

- vertical change compared to horizontal change
- change in y compared to change in x
- "rise over run"

Note that the lines and slope above do indeed match. For example:

- the line labeled with a slope of 1, does indeed have the point (2,2).
Start at (0,0). Go up 2 and to the right 2 (the positive direction).
Land on (2, 2). The slope is rise over run: up 2; over 2; 2 over 2 is 1
- the line labeled with a slope of $3/4$, does indeed have the point (3,4).
Start at (0,0). Go up 3 and to the right 4.
Land on (3,4). The slope is rise over run: up 3; over 4; 3 over 4 is $3/4$.
- the line labeled with a slope of $3/4$, also contains the point (-3, -4).
Start at (0,0). Go down 3 and to the left 4.
Land on (-4, -3). The slope is rise over run: down 4; left 3; -3 over -4 is $3/4$.

Slope As Numbers

Experiment. Predict the sound of each slope using the slope of 1 as your standard or unit.

undefined

3

2

4/3

1

3/4

2/5

0

-2/5

-1

[-2](#)

Undefined needs clarification. Slope is rise over run. One may rise 5, 4, 6, 10, but because the line is vertical, the run is 0. A constant divided by 0 is undefined.

In mountain terms, if one approaches from the bottom of a cliff so steep it can not be climbed, there is no slope because it is very, even infinitely, steep, so, no slope. If one approaches the edge of a cliff from the top, it is a very dangerous drop, so one can not safely descended, there is no slope because one would fall off, so, no slope. Rational functions like $1/x$ and $1/(x - 3)$ have x-values where the function is infinitely large. These functions have vertical asymptotes, with a slope of undefined or no slope. There can be no run. There is no slope.

If you wish, watch the [video](#) and see the last pages of the [pdf](#) file. They provide more clarity, if needed or wished.

Slope -- A Statement of How Are Things Going, A Measure of Change

Slope still needs clarification.

Slope is the [rate](#) of change. Slope states in number form how things are going, how fast they are changing and in what direction they are headed.

There is even a slope of the slope to state how the change is changing. There is even more to learn than all that.

Slope is the rate or speed at which the change is happening and the direction the change is going.

These facts are already know.

- If the amount or number or thing is getting bigger, the slope is positive, growing, increasing.
- If the amount or number or thing is staying the same, the slope is zero, unchanged, flat.
- If the amount or number or thing is getting smaller, the slope is negative, shrinking, decreasing.

Examples of "Slope is the rate of change" will help.

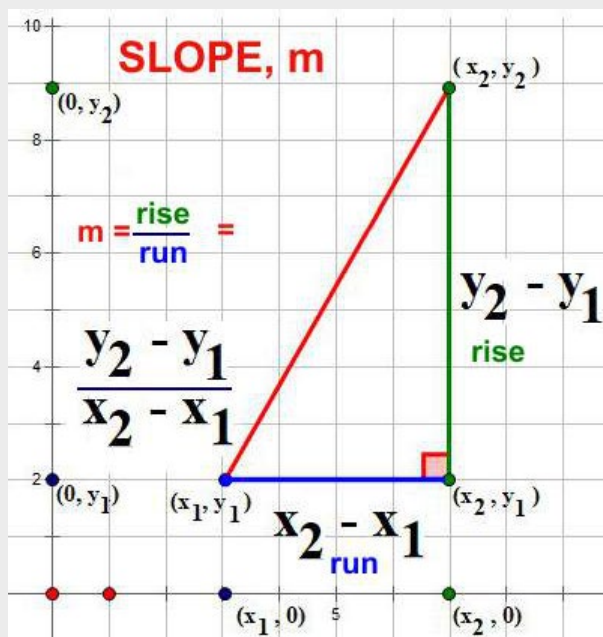
Slope may be used to compare:

- change in distance compared to a change in time, d/t
- change in speed compared change in time, v/t
- changes in size compared to change in time, $(\text{number})/t$
- change in vertical distance compared to change in horizontal distance, y/x
- change in function value compared to change in input value, $f(x)/x$

Rise Over Run with Coordinates & Slope of the Tangent

Before formal arithmetic computation of slope, a review of symbols and their meanings is a good idea. A more complete and detailed list of symbols and their meaning is found at: www.mathnstuff.com/ef/wax/symbols.gif. Here's a short but important list.

- x , the current position, location, time, number
- $f(x)$, the function value at x . the value of the thing discussed or measured at time or number x
- m , the slope at x , the rate at which the function is changing at value x
- (x, y) , a point or any old point, or a point on a curve
- (x_1, y_1) , a specific 1st point
- (x_2, y_2) , a specific 2nd point
- $m(x_1, y_1, x_2, y_2) = (y_2 - y_1) / (x_2 - x_1)$,
the slope of the line through
a first point, (x_1, y_1) , and
a second point, (x_2, y_2)
equals a ratio:
the difference in the y -values (the change in y)
compared to
the difference in the corresponding x -values (the change in x)



First point:

(,)

Second point:

(,)

Press:

for the slope.

Asides

If you wish to know more about how the sounds on this page were created, read [Slope on the HP](#). To learn how the slope graphic was produced, read [Image Map](#).



Free Geometer's Sketchpad!

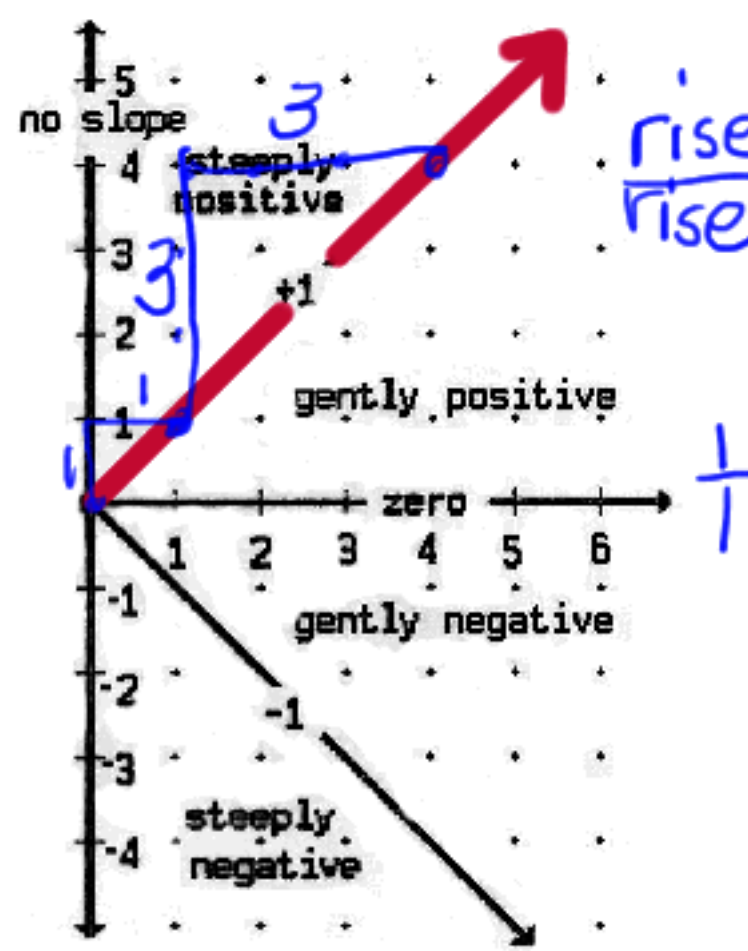
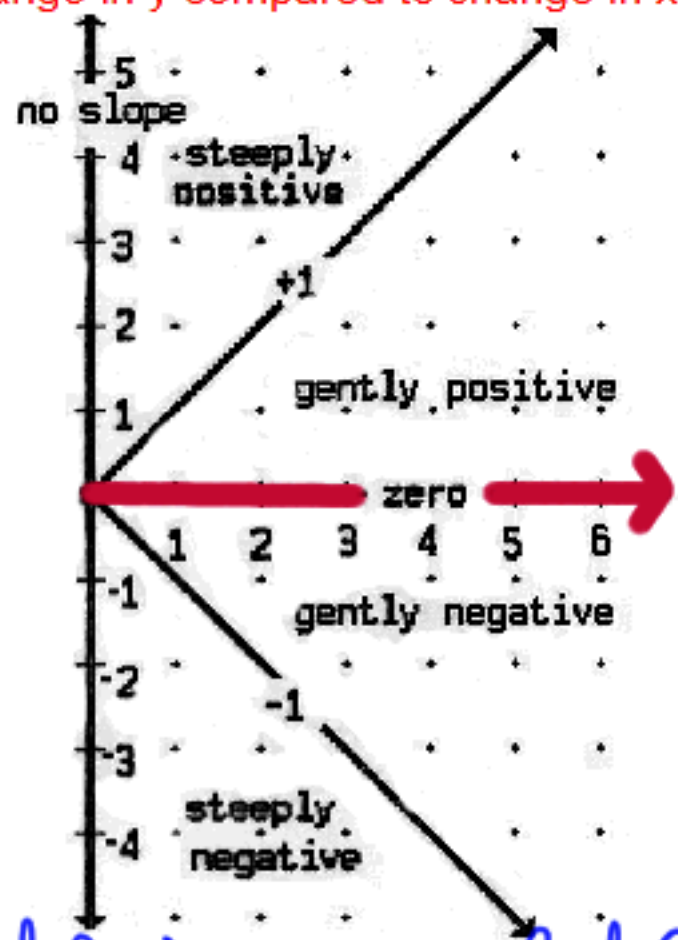
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www.mathnstuff.com/math/spoken/here/2class/240/ssong1.htm

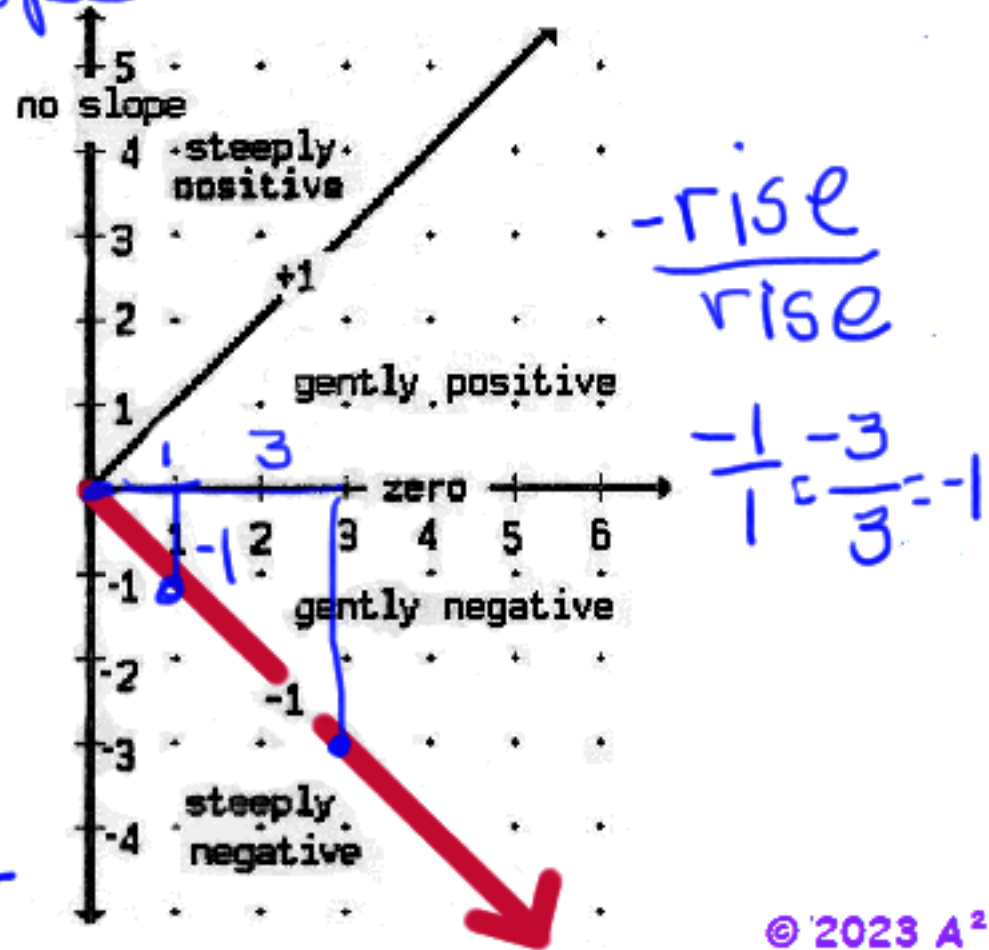
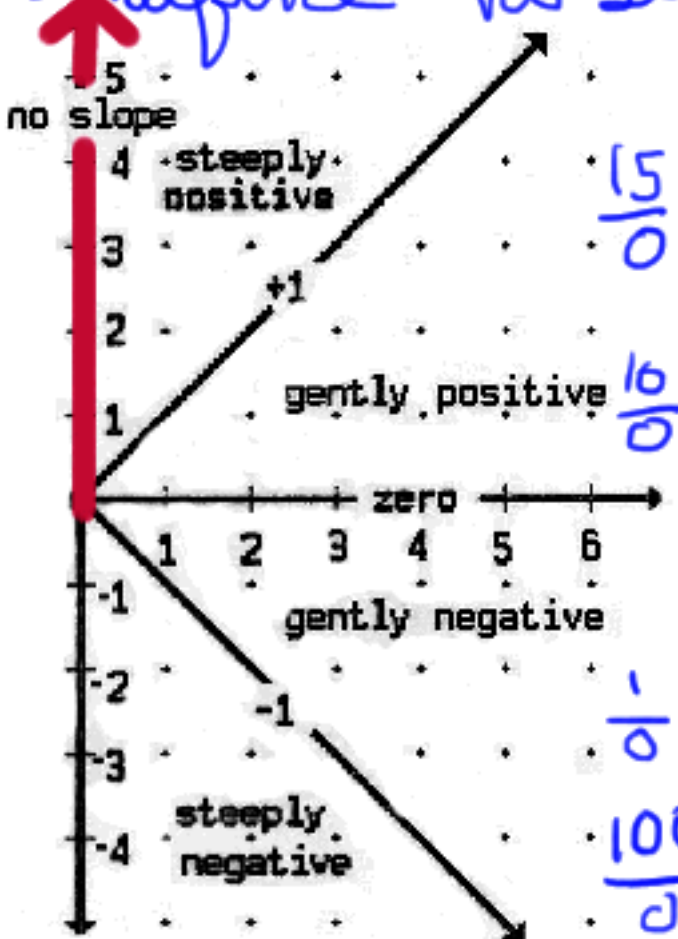
Slope is $\frac{\text{rise}}{\text{run}}$ --- "rise over run"

--- vertical change compared to horizontal change

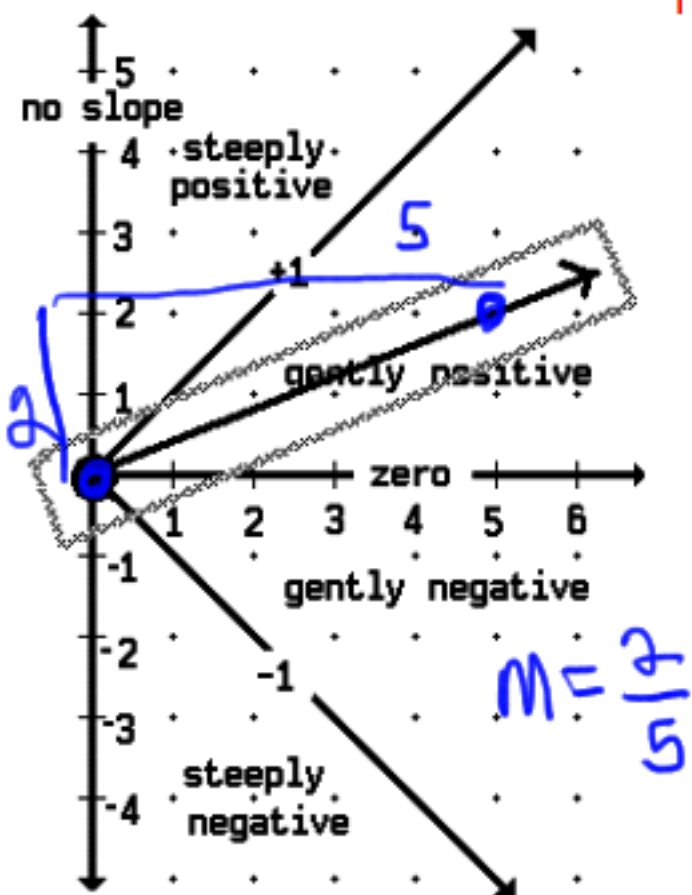
--- change in y compared to change in x



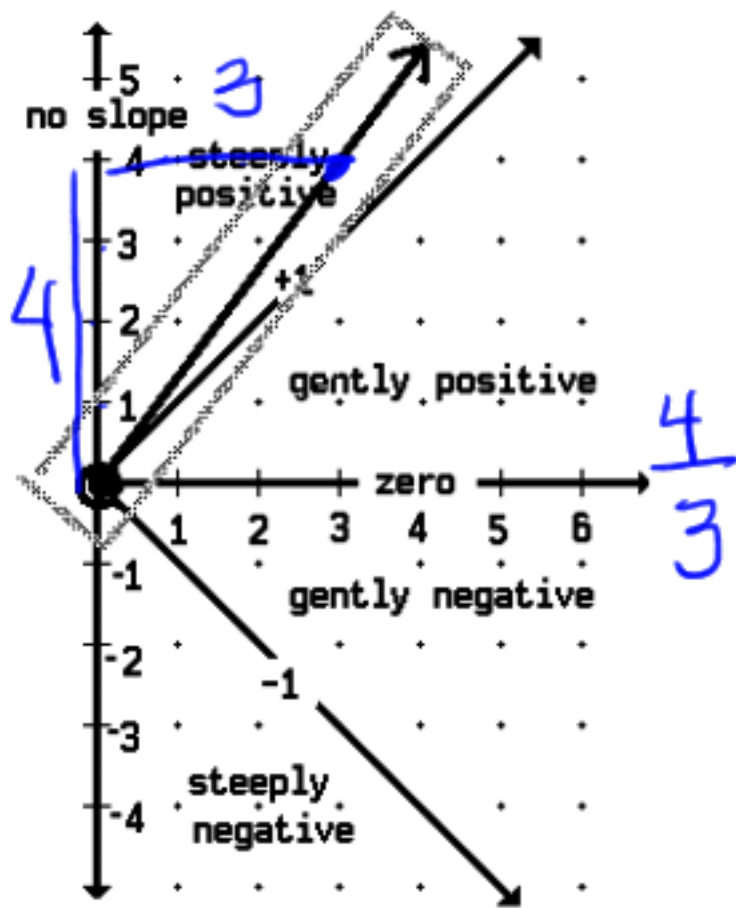
undefined no slope



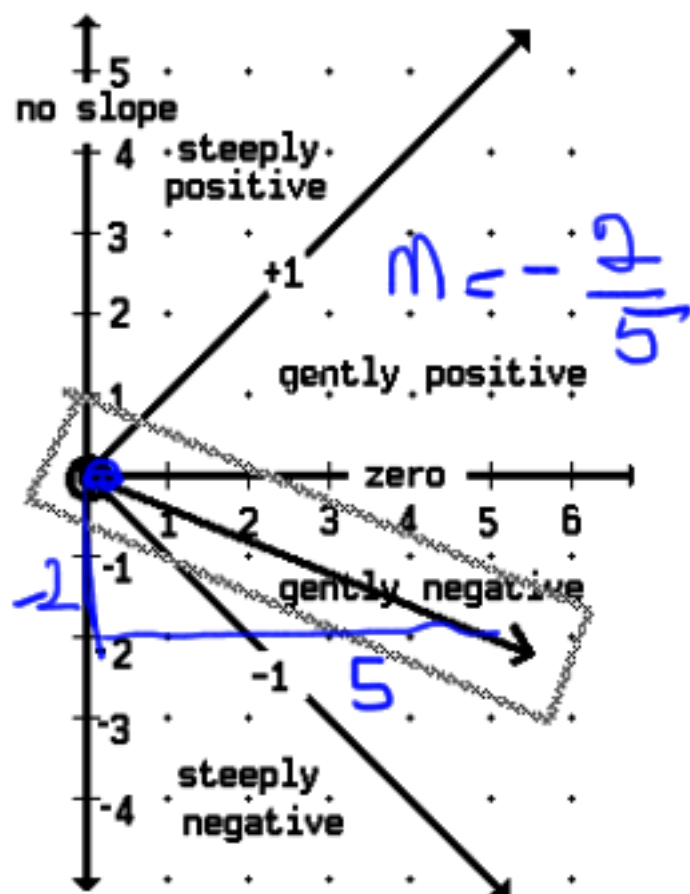
Slope is "rise over run" $\frac{\text{rise}}{\text{run}}$



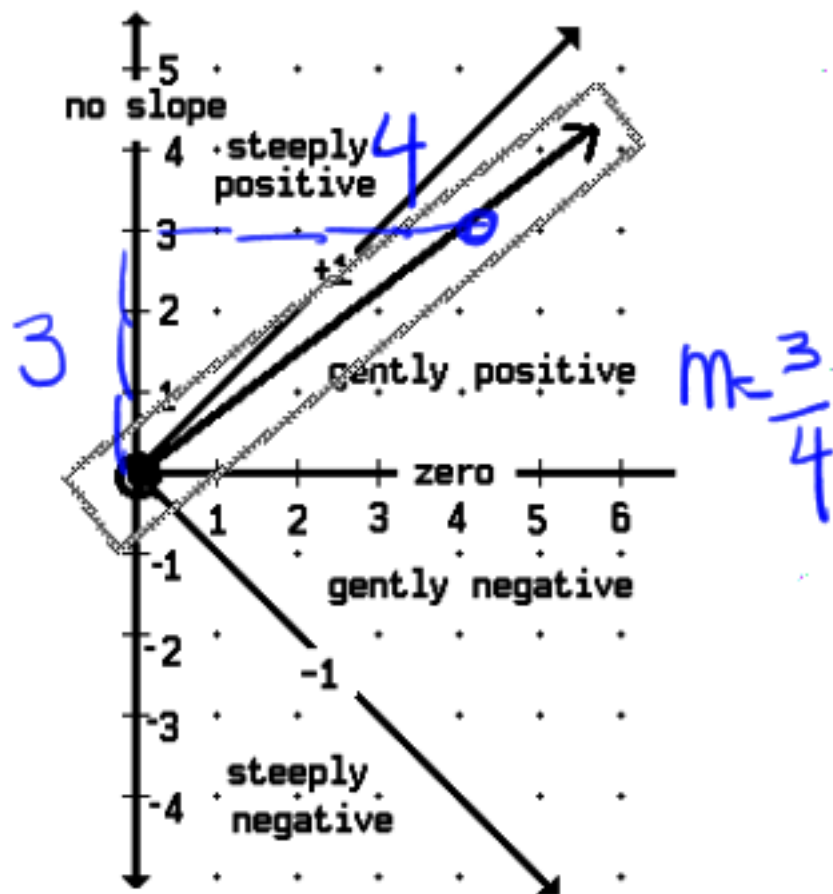
Slope is $2/5$, 2 up, 5 right.



Slope is $4/3$, 4 up, 3 right.



Slope is $-2/5$, 2 down, 5 right.



Slope is $3/4$, 3 up, 4 right. © 2023 A²

$$\text{steepest roller coaster}(x) = \left(\frac{-200}{1} \right) \cdot x + 200$$

$$\text{roller coasted}(x) = \left(\frac{-8}{9} \right) \cdot x + 200$$

$$y(x) = (1) \cdot x + 0$$

line with $m = 1$

Slope is $\frac{\text{rise}}{\text{run}}$ --- "rise over run"
 --- vertical change compared to horizontal change
 --- change in y compared to change in x

$$\text{steepest road}(x) = \left(\frac{13}{50} \right) \cdot x + 0$$

$$\text{home ramp}(x) = \left(\frac{10}{40} \right) \cdot x + 0$$

$$\text{ramp}(x) = \left(\frac{1}{12} \right) \cdot x + 0$$

- 1:12 ramps public, 5 degrees of incline.
- 3:12, 1:4 ramps at home, almost 15 degrees.
<https://www.google.com/>
- 1 : 7 roads with steep slopes
 approximate gradient of 10% or 14%
<https://www.elizabethan.notts.sch.uk>
- 13:50 world's steepest road Sonora Pass, California, USA along CA-108, 26% grade
<https://matadornetwork.com/read/steepest-highway-grades-in-the-us/>
- 8:9 Roller Coaster Restrictions
 No ascent or descent can be steeper than 80 degrees from the horizontal.
- 200:1 steepest roller coaster plunge,
 Sheikra Roller Coaster, Busch Gardens, Tampa, Florida, USA
 - 200 ft, 90° straight down
<https://buschgardens.com/tampa/roller-coasters/sheikra>