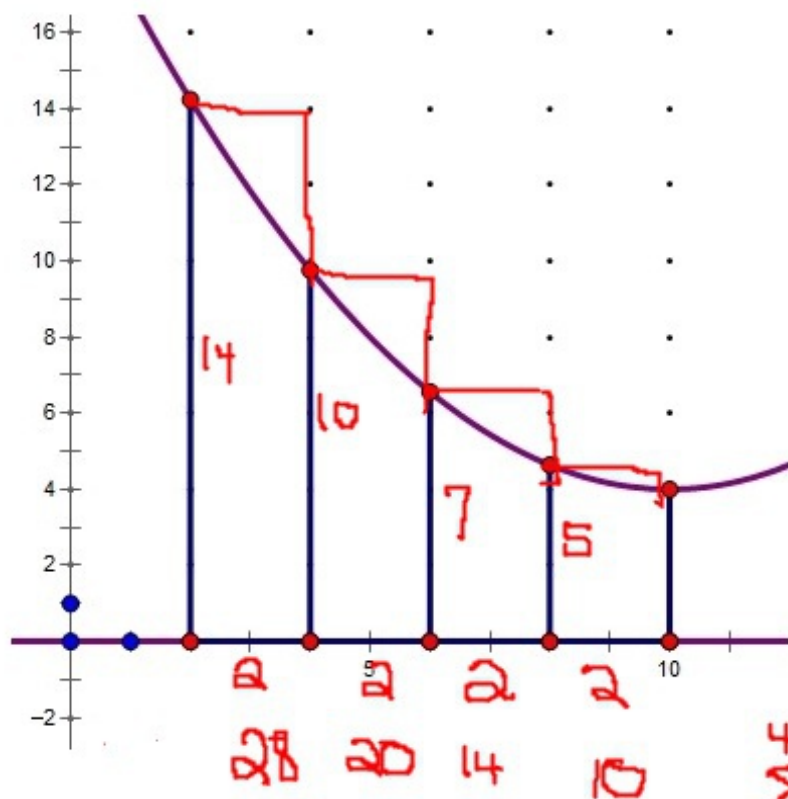


Notes on Reimann "Boxes" & Sums

Vocabulary & Symbols

real number info

- $f(x)$ -- a function defined over the Reals on the interval $[a,b]$
- $[a,b]$ -- the interval from a to b including a and b , as in $a \leq x \leq b$
- n -- the number of sub-intervals in $[a,b]$
- Δx -- the width of each sub-interval, $(b-a)/n$
- i -- interval counter, really sub-interval counter
- P -- the partition of the plane divided by the n sub-intervals
- $P = \{ [a=x_0, x_1], [x_1, x_2], [x_2, x_3], \dots, [x_{n-3}, x_{n-2}], [x_{n-2}, x_{n-1}], [x_{n-1}, x_n=b] \}$
- $(x^*i, f(x^*i))$ -- a representative x in an interval & its matching function value, height x^*i might be the
 - LEFT-most x in the interval, or the
 - RIGHT-most x in the interval, or the
 - MIDPOINT of the interval, or
 - some other chosen representative x

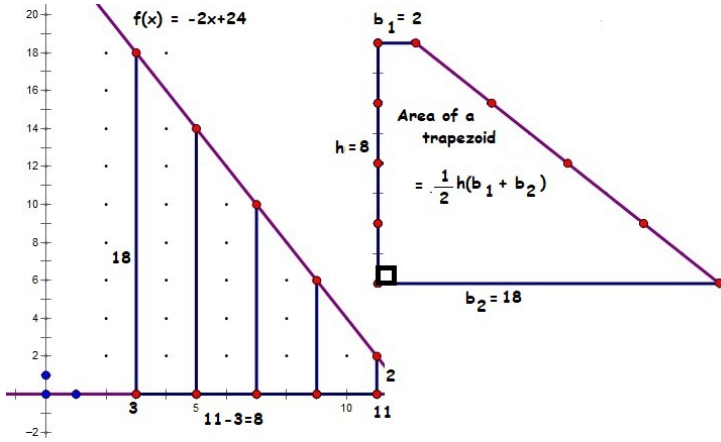


On the graph at the left,

1. State: a. $f(x)$ $f(x)$ b. $[a,b]$ $[2,10]$ c. n 4 d. Δx 2
2. Draw a rectangular box in each interval
 - a. with a width of Δx , the width of the entire interval,
 - b. with a height of the LEFT-most $f(x)$ for a LEFT Reimann sum,
 - with a height of the RIGHT-most $f(x)$ for a RIGHT Reimann sum,
 - with a height of the MIDPOINT's $f(x)$ for a MIDPOINT sum,
 - with a height of the $f(x)$ of some chosen representative x
3. For each box, compute the area of the box based on the numbers on the graph.
4. Add the areas. This is the Reimann sum.

$$\sum_{i=1}^4 f(x_i^*) \Delta x = 72$$

A Look at the Area Under $f(x) = -2x + 24$, $3 \leq x \leq 11$ from Many Points of View



1. Complete the computation for the area of a trapezoid to find the area under $f(x) = -2x + 24$, $3 \leq x \leq 11$.

$$A = (1/2)(h)(b_1 + b_2)$$

$$A = (1/2)(8)(2+18) = 4(20) = 80$$

2. Complete the computation of the definite integral to find the area under the curve $f(x) = -2x + 24$, $3 \leq x \leq 11$.

$$\int_3^{11} (-2x + 24) dx = [-x^2 + 24x]_3^{11} =$$

$$[-x^2 + 24x]_{x=11} - [-x^2 + 24x]_{x=3} =$$

$$[-(11)^2 + 24(11)] - [-(3)^2 + 24(3)] =$$

$$[-121 + 264] - [-9 + 72] =$$

$$[-143] - [63] = 80$$

3. Draw Reimann boxes for the RIGHT Reimann sum and the MIDPOINT Reimann sum. Compute the areas. Compute the sums to approximate the area under $f(x) = -2x + 24$, $3 \leq x \leq 11$.

