



The Lecture Version of

[fX.f'X.f'X.intX.gsp](#)

- Limits, Derivatives, Antiderivatives, FTC I, FTC II Presented Dynamically & Analytically

LOGISTICS

Links

- [fX.f'X.f'X.intX.gsp](#), the Sketchpad
- [pdf of this page](#)
- [video of this talk](#)
- [Teachers' Manual web page](#)

Claims and Purposes the fX.f'X.f'X.intX.gsp Sketchpad

- It focuses on the topics of differentials, Reimann Boxes, and the FTC I & II, but, has review material on limits and derivatives.
- It suggests the early introduction and use of a term like a "cumulative function," as in cumulative probability.
It suggests the use of the "Mother Tongue" rather than just "mathematics."
See [The Languages of the Math Classroom](#)
- It facilitates an analytic and a numeric approach to topics.

Disclaimers about the fX.f'X.f'X.intX.gsp Sketchpad

- It DOES NOT present an algebraic approach or deal with computation problems and solutions.
- It DOES NOT REALLY TAKE A DERIVATIVE.
It uses $[f(x + .005) - f(x - .005)] / .01$ as an approximation of the derivative.
- It DOES NOT INTEGRATE.
It uses sums of Reimann boxes plotted to produce a plot of a cumulative curve.

This talk is just a brief summary of what this Sketchpad can do. Here we BRIEFLY examine "Limits, Derivatives, Antiderivatives, FTC I, FTC II Presented Dynamically & Analytically."

LIMIT by approach

- Take a limit, as x approaches c , where $f(x)$ is continuous/not continuous.
- Take a limit, as x approaches infinity.

DERIVATIVES by definition, secant

- Slide to make h go to infinity as the slope of the secant approaches the slope of the tangent, the derivative at $(x, f(x))$.
- Use emojis to mark status of the function.
- Find & mark other values of C , $f(C)$, $f'(C)$, $f''(C)$, intervals over which the function is increasing/decreasing/zero.
- Plot the 1st, 2nd, 3rd, 4th derivative of a function using the derivatives of $\sin(x)$ as example.

PARTITION & SUMS 4 boxes

- Adjust the partitioning of an interval.
- State/show Reimann Boxes notes, left-hand boxes, right-hand boxes, midpoint boxes areas, sums
- Show int, from a to b , $[(f(x)) dx]$ using the "hump areas" of sine/cosine.
- Show int, from a to b , $[(f(x)-g(x)) dx]$
- Show cumulative probability distribution.

HISTORY

- Note how a finite sum becomes an integral.
- State history of symbols & vocabulary.
- State & explain FTC I & FTC II.

INTEGRATION by dots

- Examine how the plotted points & how curve is off by a constant.
- Examine integral of $f(y)$

The talk ends here.

Below are listed the Links and Activities on each sheet of the Sketchpad as found in the Teachers' Manual web page

Review If Needed

0 - toc

Links

[Functions, graphing, fancy, history \(see 0 - toc\)](#)

[Derivatives Web Page](#)

[Reimann Sums gsp](#)

[ReimannSumNotes.pdf](#)

[Intro to Antiderivatives](#)

[Derivative Calculator](https://www.derivative-calculator.net/) at <https://www.derivative-calculator.net/>

[Integral Calculator](https://www.integral-calculator.com/) at <https://www.integral-calculator.com/>

Activities

* Use the above pages & calculators to assist in writing a test.

1 - LIMIT by approach

Links[endbehavior.htm](#)[limit.htm](#)[limit.gsp](#)**Activities**

1st. Slide the x value to approach.

- * Take a limit, as x approaches c, where $f(c)$ is continuous
 - * Take a limit, as x approaches c, where $f(c)$ is not continuous, as in $x=c$ is a vertical asymptote
 - * Take a limit, as x approaches infinity
 - * Change the function & repeat the above
 - * Examine endbehavior
-

2 - DERIVATIVE by definition, secant

Links[limit.htm](#) which includes [limit.gsp](#)

Show work

Activities

- 1st. Drag the red point to make h smaller, closer to 0, to make h approach 0, to obtain the derivative.
 - 2nd. As h gets smaller the secant line EF becomes more like a tangent line.
 - 3rd. Try to slide the red point so close to $(x, f(x))$ that the slope of the secant equals the slope of the tangent, the derivative at $(x, f(x))$.
-

3 - DERIVATIVE by m of tangent line, x

Links[TABLE f, f\(x\), f'\(x\), f''\(x\) DerAnyFx.gsp then page 5](#)also [See No. 20 below regarding "Derivative TABLE found in DerAnyFx.gsp"](#)

Show computation & functions

Activities

- * Use Ctrl + C to make more emojis as needed.
 - * Use emojis to mark status of the function.
 - * Find & mark the zeros of the function.
 - * Find & mark the zeros of the first derivative.
 - * Discuss the status of the function at these points/values.
 - * Find & mark the zeros of the second derivative.
 - * Discuss the status of the function at these points/values.
 - * Find & mark other values of C, $f(C)$, $f'(C)$, $f''(C)$.
 - * Discuss intervals over which the function is increasing/decreasing/zero.
 - * Discuss intervals over which the function is concave up/down.
 - * Summarize as desired.
 - * Change the function & repeat the questions/activities.
-

4 - DERIVATIVE by trace

Links

Hide/show trig memory trick

Hide/show derivative functions

Hide/show Teaching Activities.

Hide/show 1st derivative in green

Hide/show 2nd derivative in purple

Hide/show 3rd derivative in orange

Hide/show 4th derivative in red

Activities

Trace the derivatives.

1st: Turn OFF then ON in Display Menu "Trace Point" OR use Ctrl + T.

2nd: Drag the DOT ON THE AXIS to trace that color derivative.

3rd: Erace the trace with Display Menu OR Shift + Ctrl + E.

4th: Trace the derivatives.

Teaching activities

1. Enable the tracing, trace, name the dot-drawn derivative, record it on the screen w/pen
2. Repeat step 1 with second derivative.
3. Repeat step 1 with the next derivative.
4. Repeat step 1 with the next derivative.
5. Reflect on/Discuss the result of all graphs.
6. Unhide the memory trick.

Partition, Differential, Boxes

5 - PARTITION & SUMS 4 boxes

Links

Hide/Show Boxes

Hide/Show $g(x)$

Hide/Show left-hand sums

Hide/Show $g(x)$

Go to [ReimannSums.gsp](#)

Notes

- * [Reimann.htm](#) exists and is formatted like this page and includes [ReimannSumNotes.pdf](#) and [ReimannSums.gsp](#)
- * The next page shows example of area and sum computation.

Activities

- * Show boxes.
- * Discuss and position left-hand boxes, right-hand boxes, midpoint boxes.
- * Discuss the areas and sum of area of left-hand boxes, right-hand boxes, midpoint boxes.
- * Move boxes from picture.
- * Change a and b and perhaps $f(x)$.
- * Show boxes and discuss areas.
- * Move boxes from picture.

6 - REIMANN & SUMS

Links

* [ReimannSumNotes.pdf](#)

* [Reimann.htm](#) including [ReimannSums.gsp](#)

Activities

- * Discuss areas, summation meaning & format
- * Perhaps go to links

7 - SUMS $f(x) - g(x)$, **slide end points to change [a, b]**

Links

Show height and area computation

Show midpoint coputation

Show $g(x)$

Activities

1. Start w/ $g(x) = 0$
2. Ask can an area be negative?
- * Show negative & positive areas.
- * Show the left vs midpoint vs right turning negative as x is dragged.

3. Ask how would increasing the number of boxes effect the area?
 4. Ask which is the best approach for computation & why?
 5. Reveal $g(x)$ and discuss height.
 6. Change $f(x)$, $g(x)$, or both.
 7. N is small (so the differences in the 3 different sums is large).
Use the midpoint sum to answer this question.
As is, the sum is an approximation of the area under $f(x)$ from a to b .
What is the result if one wishes the sum from b to a ?
-

8 SUMS $f(x)$ - $g(x)$, **input [a,b] in the boxes for a and b**

Note

This page only provides the left-sum, but provides a more accurate way to name a and b .

Links

Show $g(x)$

Activities

- * Change $f(x)$ to an odd function and ask questions.
 - * Change $f(x)$ to an even function and ask questions.
-

9 - CUMULATIVE AREA probability distribution

Notes

My calc students don't usually take statistics because they are taking calc.
This sheet serves as a background and shows a real life example of using an integral.
 N is 32. This is small, compared to infinite, but large enough to provide a good estimate.

Two images are provided. The top is what the sheet usually looks like.
The bottom shows notes.

This also illustrates the use of the term "cumulative" to indicate the integral of a function and provide/state cumulative results.

Links

- * Show function $f(x, \mu, \sigma)$
- * Show constant 1.
- * Show constant 2.
- * Download statistics spreadsheet - use sheet cum
This has an integration function feature.
- * Link to "Probability for Calc I"
This provides a summary of statistics and the normal probability distribution.

Activities

- * Play.
-

10 - HISTORY sum 2 integral

Links

- 1st. Symbols & vocabulary
- 2nd: Exactly how do they relate?
- 3rd: How Does the finite go to the infinite?
- 4th: FTC I and II

Activities

- * In order reveal and discuss the notes.
-

11 - INTEGRATION by dots

Links

FTC I and II

Plots on the next page

Activities

- * In order reveal and discuss the notes.

12 - INTEGRATION FTC I & FTC II

Notes: The plotted points are off by a constant. It is an indefinite integral. Change the parameter "plus c" to adjust the plotted antiderivative points.

Links

- * Show $g(x)$ and height = $h(x)$
- * Show ordered pairs of plotted points.
- * Show "Why plus c?"
- * Show blue Suggested Functions box.

Activities

- * Change the function as desired.
- * Functions that fit on the screen are suggested.

Extras

13 - INTEGRATION $f(y)$

Links

Activities

- * Play.

20 - Derivative TABLE found in DerAnyFx.gsp

Links

[DerAnyFx.gsp](#) -- derivatives of any function -- includes building a TABLE

Activities

- 1st: Drag the blue point to change x and show a new tangent line.
- 2nd: Below, select, left to right each of the measurements.
- 3rd: In Number Menu, select Tabulate and a table is created.
- 4th: With Tabulate selected, click once and go to Properties.
- 5th: Drag the blue point to a new x value (abscissa).
- 6th: Double click on table to add another row with the new abscissa.
- 7th: Repeat as desired.
- 8th: To delete a row, Shift + Click + click.

www.mathnstuff.com/math/gsp/sumr19/su19newgsp/fX.f'X.f'X.intX.gsp



www.mathnstuff.com/papers/fX.intX.8.15.22.htm

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